Official Sovereign Debt

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Introduction

- Much of the emerging markets sovereign debt is from official lenders
  (Schlegl-Trebesch-Wright 2019)
  - Official lenders: bilateral governments and multilateral organizations
  - Flows in during disasters – wars, natural, financial (Horn-Trebesch-Reinhart 2020)
- Debt tends to increase during sovereign defaults
  (Arellano-MateosPlanas-RiosRull 2023, Benjamin-Wright 2009)

What is the role of official debt during sovereign defaults?

Can official debt be used to improve resolutions of sovereign defaults?

What we do

- Document patterns of official and private debt during defaults in emerging markets
  - Official debt is large and flows in during sovereign defaults
- Framework of sovereign partial default with official and private debt
  - Official debt: longer maturity and more concessional (lower recoveries)
  - Sovereign can default: default does not eliminate debt nor precludes borrowing
  - Longer maturity better for debt capacity, more concessional worse for debt capacity
  - Can rationalize much of the patterns
- Counterfactuals: voluntary swaps of private for official during defaults is welfare improving

Literature

- Official lending in sovereign debt markets empirically

  (Horn, Reinhart, and Trebesch 2020, 2021)

- Multilateral lending and China lending

  (Boz 2011, Kirsch-Rühmkorf 2017, Roch-Uhlig 2018, Kondo-Mkhitaryan-SosaPadilla 2022)

- Default risk and maturity (Arellano-Ramanarayanan 2012, Hatchondo-Martinez-SosaPadilla 2016, Bocola-Dovis 2019, Aguiar-Amador-Hopehayn-Werning 2019)
  - Short debt better for commitment; does not feature dilution problem
  - Long good for rollover crises
  - The focus is on pre-default

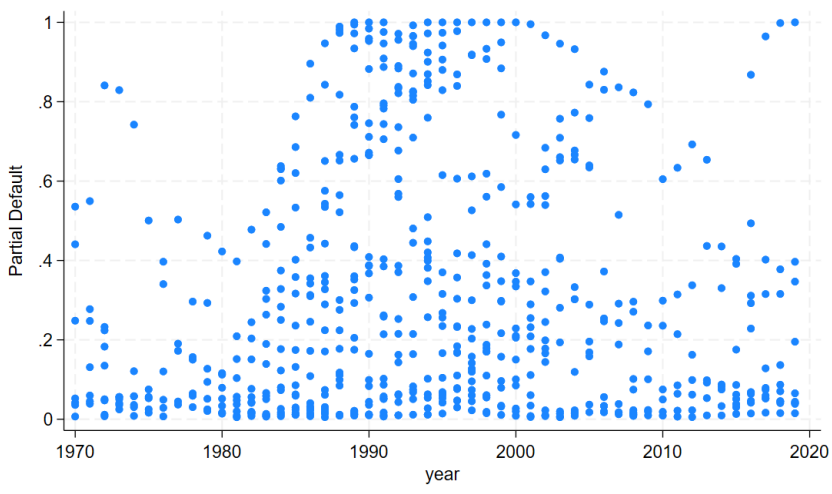
- Partial defaults, increased debt and maturity extensions in default

  (Arellano-MateosPlanas-RiosRull 2023, Benjamin-Wright 2009, Dvorkin-Sanchez-Sapriza-Yurdagul 2019, Mihalache 2020)

Here long-term debt more debt capacity: rationalizes official long-term debts during defaults

Partial Default Over Time and Countries

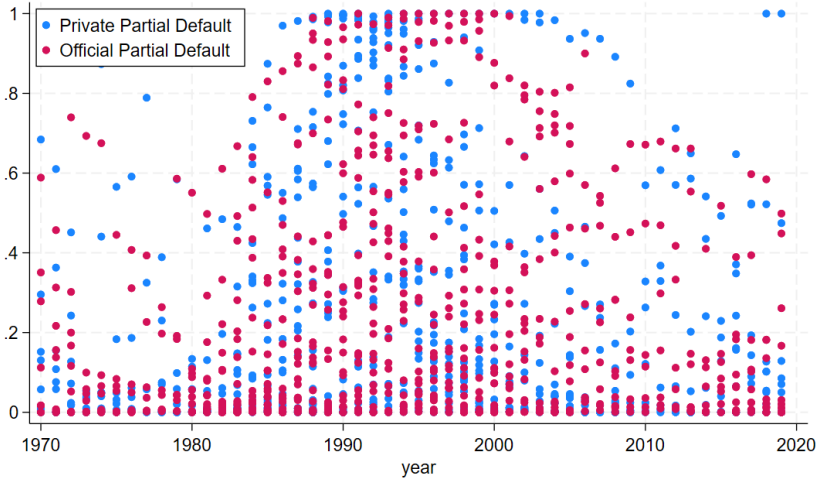
30 countries, 50 years



- Partial default (debt in arrears/ debt due) varies widely, mean 32% and st. dev. 24%

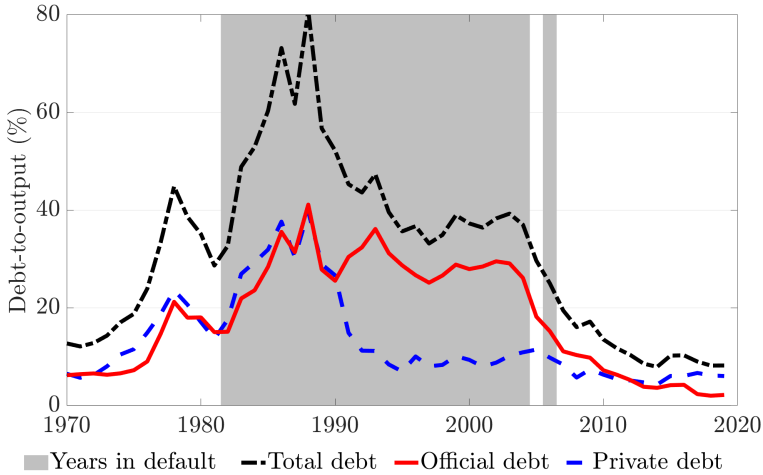
Partial Default: Private and Official

30 countries, 50 years



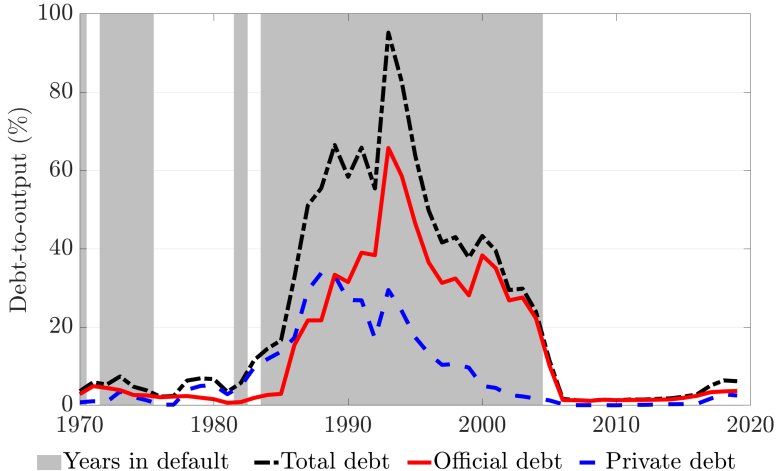
- Partial default on private and official debt correlated = 72 %

Official and Private Debt in Peru



- Official debt accounts for much of the debt at the end of the default episode

Official and Private Debt in Nigeria



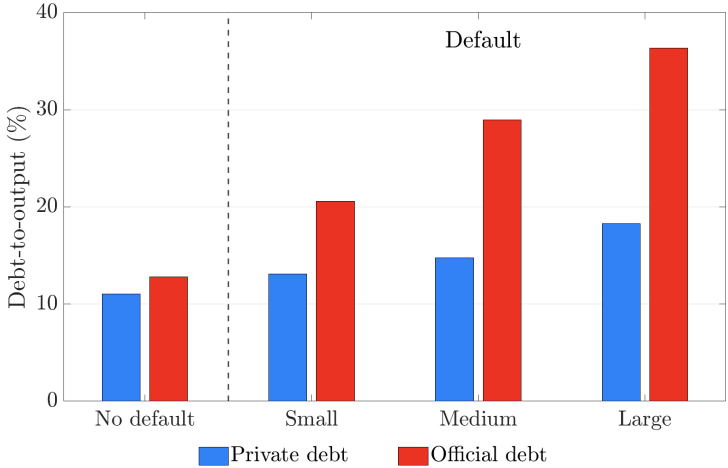
Debts during Defaults

30 countries, 50 years

	No default	Partial default
Partial default	0	32
Debt to output (in %)		
Total	23	44
Official	13	29
Private	11	15
Spreads	4	11
Output	2	-3

- Partial defaults associated with higher debt, spreads, and lower output
- Official debt more than doubles during defaults, private increases only moderately
- Default episodes last on average 10 years, recovery 20% for official and 60% for private

Debts during Default



- Official debt flows in during defaults, more so in severe default

Model Environment

- Small open economy with stochastic endowment z_t that borrows internationally
- Borrows long-term from official and private lenders
  - Terms of debt contract depend on type of debt
- Can default on both types of debts selectively
- Sovereign chooses loans and defaults for the economy
- Prices of bonds compensate lenders for loses from default

Debt Contracts

- Debt contracts perpetuities with decay ϑ^i
- Each contract a_t^i has a coupon due and the sovereign can partially default on it

$$(1 - d_t^i) \quad \underbrace{a_t^i R^i}_{\text{coupon due}}$$

- Accelerates default on μ^i of the legacy debt

$$a_{t+1}^i = (1 - \mu^i d_t^i)\vartheta^i a_t^i \ + ...$$

(acceleration clauses allow future coupons to be in default)

- Defaulted coupons accumulate as future debts with recovery factor κ^i

$$a_{t+1}^i = (1 - \mu^i d_t^i)\vartheta^i a_t^i + \kappa^i d_t^i a_t^i \ + ...$$

(loans with large write-offs have low $\kappa^i = \hat{\kappa}^i(R^i + \mu^i\vartheta^i)$, more concessional in default)

- Official and private debt differ in: duration ϑ^i, acceleration μ^i, and concessional κ^i

Sovereign Borrower

- Preferences over consumption $E \sum_{t=0}^{\infty} \beta^t u(c_t)$
- Consumption is income y_t net of repayment of debt service and new borrowings ℓ^i

$$c_t = y_t - \sum_{i=f,b} (1 - d_t^i) a_t^i R^i + \sum_{i=f,b} q_t^i \ell_t^i$$

- Laws of motion for debts: legacy debts, accumulation of defaulted debt, new borrowings

$$a_{t+1}^i = (1 - \mu^i d_t^i) \vartheta^i a_t^i + \kappa^i d_t^i a_t^i + \ell_t^i$$

- During defaults income is lower: $y_t = z_t \psi(d_t^f, d_t^b, z_t) \leq z_t$
- Sovereign can always borrow, even with default, but prices q_t^i respond

Value and Bond Prices Functions

- Let $a^i = (f, b)$: $V(f, b, z) = \max_{\ell^f, \ell^b, d^f, d^b} \{u(c) + \beta \, \mathsf{E}_z \, V(f', b', z')\}$

  subject to budget constraint, laws of motion for debts

- No separate problem in default, partial default a period by period decision

- Bond prices compensate lenders for default losses for each type of debt

$$q^i(f', b', z) = \frac{1}{1+r} \, \mathsf{E} \left[(1 - d^{i\prime}) R^i + \left(\kappa^i d^{i\prime} + \vartheta^i (1 - \mu^i d^{i\prime}) \right) q^i(f'', b'', z') \right]$$

- Default next period + value of accumulated arrears + future coupons

Characterization of Partial Default

- Given states and potential debt choices (b, f, b', f'), choose partial defaults to max

$$c = z\psi(d^b, d^f, z) - (1 - d^b)bR^b + q^b\ell^b - (1 - d^f)fR^f + q^f\ell^f$$

$$\ell^b = b' - b\vartheta^b + d^b(\kappa^b - \mu^b\vartheta^b)b$$

$$\ell^f = f' - f\vartheta^f + d^f(\kappa^f - \mu^f\vartheta^f)f$$

- Partial default on each type of debt $i \in \{f, b\}$ chosen to expand the budget

$$-\psi_{d^b}(d^f, d^b, z) = \frac{b}{z}[R^b - q^b(\kappa^b - \mu^b\vartheta^b)]$$

$$-\psi_{d^f}(d^f, d^b, z) = \frac{f}{z}[R^f - q^f(\kappa^f - \mu^f\vartheta^f)]$$

Characterization of Partial Default

$$-\psi_{d^b}(d^f, d^b, z) = \frac{b}{z}[R^b - q^b(\kappa^b - \mu^b \vartheta^b)]$$

- LHS, marginal costs of partial default for output losses
- RHS, marginal benefits from expansion of resources from default: coupon savings bR^b - net accumulated arrears evaluated at market prices $bq^b(\kappa^b - \mu^b \vartheta^b)$
- High debt b to z, low bond prices q^b increase default incentives
- Gives decision rules for partial default: $\mathrm{d}^i(f, b, z, f', b')$

Portfolio Decision

For simplicity assume, $\kappa = \mu = 0$

$$u_c\left(q^b + \frac{\partial q^b}{\partial b'}(b' - \vartheta^b b) + \frac{\partial q^f}{\partial b'}(f' - \vartheta^f f)\right) = \beta E u_c'\left((1 - d^{b\prime})R^b + \vartheta^b q^{b\prime} - \frac{\partial d^{b\prime}}{\partial b'}(z'\psi'_{d^b} + R^b b') - \frac{\partial d^{f\prime}}{\partial b'}(z'\psi'_{d^f} + R^f f')\right)$$

$$u_c\left(q^f + \frac{\partial q^f}{\partial f'}(f' - \vartheta^f f) + \frac{\partial q^b}{\partial f'}b' - \vartheta^b b)\right) = \beta E u_c'\left((1 - d^{f\prime})R^f + \vartheta^f q^{f\prime} - \frac{\partial d^{f\prime}}{\partial f'}(z'\psi'_{d^f} + R^f f') - \frac{\partial d^{b\prime}}{\partial f'}(z'\psi'_{d^b} + R^b b')\right)$$

- Increase borrowing if: price is high, elasticity of prices w.r.t. debt low, future expected repayment low, and marginal default cost is low
- Relative elasticities of bond prices w.r.t. debts and default costs key for portfolio
- Model has the same forces for borrowing incentives for periods of high default

Simple Economy Characterization

- Show that longer-term debt gives greater debt capacity
- Different from standard full default theory: short-term debt associated more debt capacity (related to Aguiar-Amador-Werning-Hopenhayn 2019 and Arellano-Ramanarayanan 2012)

Simple Economy

- Consider $u(c) = c \geq 0$, $\vartheta^f = 0$, $\vartheta^b = 1$, $\kappa^i = \mu^i = 0$ for all i, and $(1 + r)\beta < 1$
- Absent default, constant output $z_t = z$. Falls to z_L if $d_{f,t} > 0$ or $d_{b,t} > 0$.
- Key differences with standard model: market access during default + partial default (default only on coupons)

Simple Economy Characterization

- Debt capacity depends on default incentives
- Default is binary: $d^b = d^f = \{0, 1\}$

  Consumption with repayment $\quad c = z - rf - (1+r)b + q^f(f', b')(f' - f) + q^b(f', b')b'.$

  Consumption with default $\quad c = z_L + q^f(f', b')(f' - f) + q^b(f', b')b'$

- Default policy: $d^b = d^f = 1 \quad$ if $\quad rf + (1+r)b \geq z - z_L$

Simple Economy Characterization

- Default policy: $d^b = d^f = 1$ if $rf + (1+r)b \geq z - z_L$
- Suppose no initial debt $b_0 = f_0 = 0$

Only Private Loans: Repayment commitments for 1 period ahead

- Maximum private loan: $b_{\max} = \frac{z - z_L}{1+r}$ with $q^b = 1$
- Given linearity and impatience, optimal to maximize consumption at $t = 0$

$$c_0 = z + \frac{z - z_L}{1+r}$$

- Committed to repay $(1+r)b_{\max}$ the next period, but otherwise no further commitments

$$c_t = z - (1+r)b_{\max} + q^b b_{t+1} = z_L + q^b b_{t+1} = z_L + \frac{z - z_L}{1+r} \qquad \forall \quad t \geq 1$$

- Can borrow more from $t \geq 1$ which keeps consumption elevated

Simple Economy Characterization

- Default policy: $d^b = d^f = 1 \qquad \text{if} \qquad rf + (1+r)b \geq z - z_L$

Only Official Loans: Repayment commitments for all periods ahead

- Official loan maximizes budget $f_{\max} = \frac{z - z_L}{r}$; borrow to the max at $t = 0$ with $q^f = 1$

$$c_0 = z + \frac{z - z_L}{r}$$

- Consumption low for $t \geq 1$ to pay for future coupons, no more loans $f_{t+1} - f_{\max} = 0$

$$c_t = z - r f_{\max} + q^f (f_{t+1} - f_{\max}) = z_L$$

- Long-term debt with acceleration clauses has debt capacity of short debt

Lemma

Official loans expand the budget set more than private loans

$$q(f'_{max}, b' = 0) f'_{max} = \frac{z - z_L}{r} > q(f' = 0, b'_{max}) b'_{max} = \frac{z - z_L}{1 + r}$$

Quantitative Analysis

- Parameterize model to panel data of official and private debt and partial default
- Illustrate debt dynamics and partial defaults
- Evaluate performance for debts during partial defaults
- Counterfactuals and welfare:
  - Room for voluntary swaps of private and official debts
  - Official debt tends to increase welfare
  - Best design for official debt: longer and less concessional

Parameter Settings

- Estimate 8 parameters to match 10 moments: properties of debts, debt service, partial default, output volatility
- Default cost function: symmetric, convex in default, fixed cost

$$y = z\ (1 - \gamma\ d_b^2)(1 - \gamma d_f^2)(1 - \phi \mathcal{I}_{d,\hat{z}>0})$$

- Other parameters set from literature+data: risk free rate $r = 0.02$, risk aversion coefficient $\sigma = 2$, z persistence $\rho = 0.87$, acceleration $\mu^i = 0.18$, R^i normalization

Debt contracts	
Decay parameters	$\vartheta_f = 0.91$, $\vartheta_b = 0.79$
Recovery factor	$\kappa^f = 0.11$, $\kappa^b = 0.19$
Default Costs	
Based on partial default	$\gamma = 0.06$
Asymmetric endowment	$\phi = 0.8$
Discount factor	$\beta = 0.954$
Output volatility	0.052

	Duration	Recovery
Official debt:	9 year	40%
Private debt:	4.5 year	51%

Moment Matching Exercise

	Data	Model
Total Debt	33	34
Official Debt	20	21
Private Debt	13	13
Partial Default	32	28
Official debt service	1.6	1.7
Private debt service	1.9	2.3
sd(Total Debt)	18	18
sd(Official Debt)	12	12
sd(Private Debt)	8	6
sd(Output)	11	12

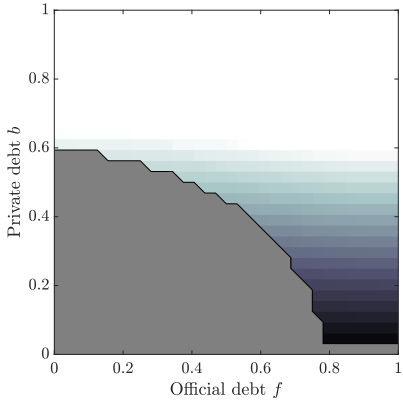
8 parameters to target 10 moments

$$\text{Partial default} = \frac{d^b b\, R^b + d^f f\, R^f}{b\, R^b + f\, R^f}$$

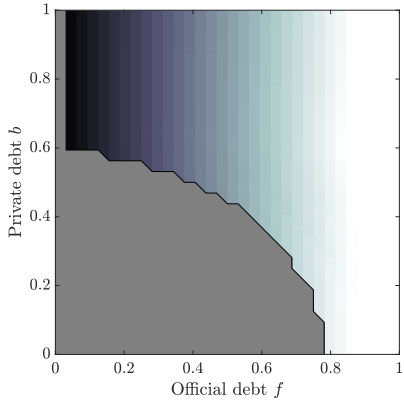
- Partial default informs default costs
- Debt services inform debt duration
- Mean and volatility of official and private debts inform recoveries and durations
- Means and volatility of total debt, β

Default Decision Rules

Private default



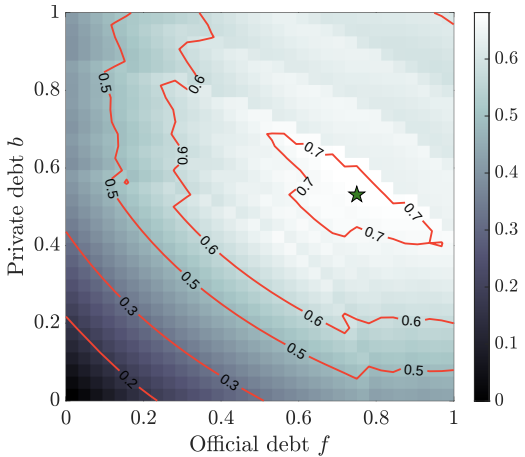
Official default



- No default for lower debts, high default for high debt
- Official debt higher debt capacity: can borrow without default up to 0.8 official and 0.6 private
- More default when portfolio tilted to one type of debt

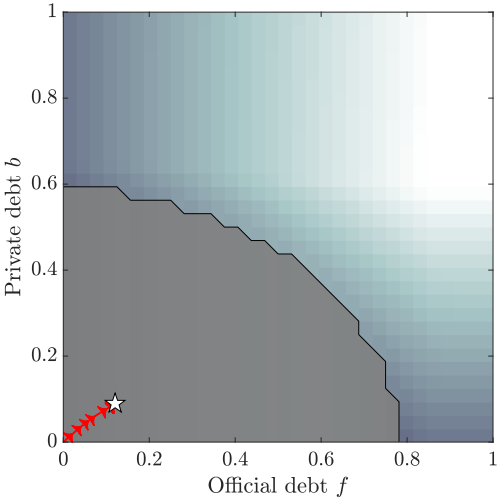
Bond Prices

total resources borrowed $q^b(f,b,z)b + q^f(f,b,z)f$



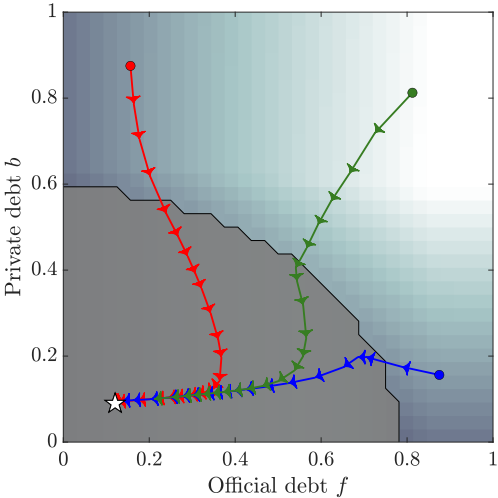
- Defaults and dynamics shape bond prices
- Value of debt across state space $q^b(f,b,z)b + q^f(f,b,z)f$
- Higher (f,b) more resources but capped by peak (star) – star more tilted towards official
- Various portfolios (f,b) give same resources: portfolios tilted towards official need less private debt to reach a level of resources
- Shapes of functions affect debt dynamics

Debt Dynamics Paths



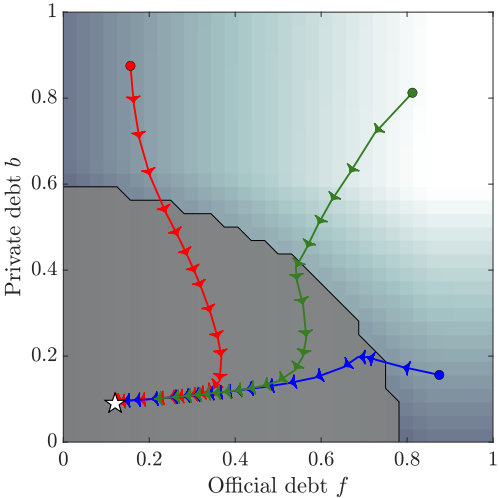
- Transition for y_M: increases both debts, frontload consumption, no default
- "Steady State" based on elasticity of bond prices w.r.t. debt
- Official level higher because of higher debt capacity
- No default for y_M, but other levels with positive default

Debt Dynamics Paths: Exiting Default



- Equilibrium exit from default by reducing debts
- Official loans used to reduce private debt faster
- Portfolio used actively to reduce consumption costs of deleverage
- Steady state positive default probability: private spreads close to 2%

Debt Dynamics Paths: Exiting Default



- Contrast with Aguiar, Amador, Hopenhayn, Werning 2019:

  economy "takes the short route" to exit a crisis zone with positive default probability as long-term debt worse due to dilution

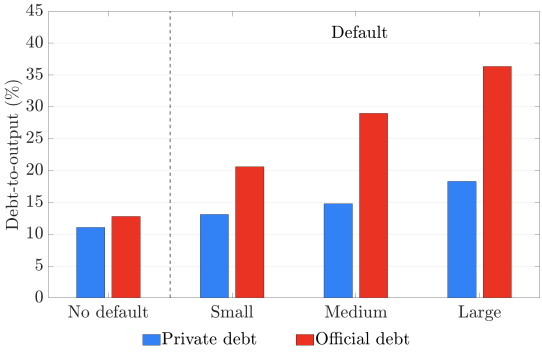
- Here use both debts to deleverage and long-term debt better for debt capacity

Moments Conditional on Partial Default

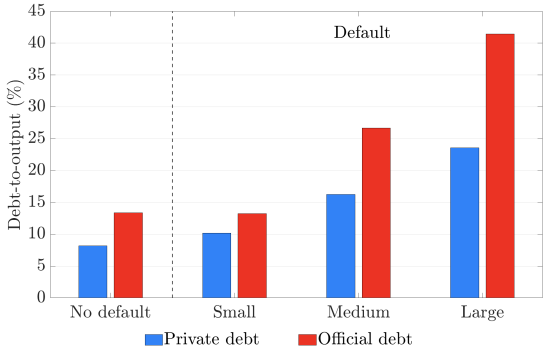
	Data		Model	
	No default	Partial default	No default	Partial default
Debt to output	24	44	21	44
Official	13	29	13	27
Private	11	15	8	17
Private spreads	4	8	1	5
Partial default	0	32	0	28

- During defaults, debt increases and spreads rise
- Official debt increases by more

Debts during Default



Data



Model

- Official debt flows in during defaults, more so in severe defaults in model and data

Dynamics During Default Episodes

	Dynamics of Debt			
	Before	Beginning	Middle	After
Data				
Total	33	35	40	33
Official	17	18	24	19
Private	16	17	17	14
Model				
Total	29	32	37	33
Official	18	20	23	21
Private	11	12	14	12

- Total debt hump shape during default episode
- Official debt accounts for most of it, still elevated at the end
- Default episodes on average 10 years in model and data

Voluntary Swaps

- Official loans have more debt capcity and official debt grows during defaults
- In baseline model each lender contracts independently and dilution effects
- Room for Pareto improvement with swaps of private and official (Brady Plan)
- Consider a state $\{b, f, y\}$. A candidate voluntary swap to $\{\hat{b}, \hat{f}, y\}$ is feasible if

$$\begin{aligned} \text{Country Welfare} \quad & V(\hat{f}, \hat{b}, z) \geq V(f, b, y) \\ \text{Lenders Total Value} \quad & H(\hat{f}, \hat{b}, z) \geq H(f, b, z) \end{aligned}$$

  with $H(f, b, z) = \hat{q}^b(f, b, y)b + \hat{q}^f(f, b, y)f$

- Consider a small deviation db', df', swaps conditions $V_b db + V_f df > 0$ and $H_b db + H_f df > 0$

Voluntary Swaps

Why $V_b db + V_f df > 0$ and $H_b db + H_f df > 0$ arise in equilibrium? (Hatchondo-Martinez-SosaPadilla 16)

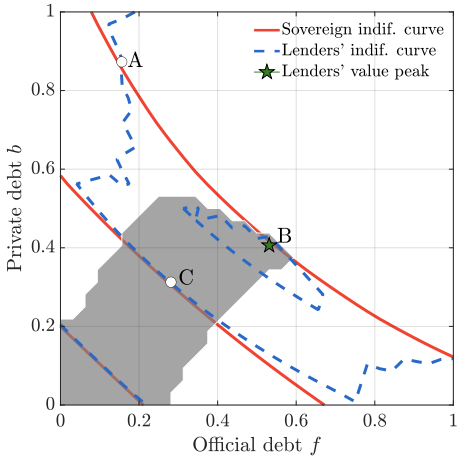
- Deviations around optimal choices satisfy this portfolio equation

$$E\ R\ \underbrace{\left(H_{b'} db' + H_{f'} df'\right)}_{\text{gain lenders}} + \frac{\beta}{u_c}\ E\ \underbrace{\left(V_{b'} db' + V_{f'} df'\right)}_{\text{gain sovereign}} = \underbrace{\vartheta^b b \left(\frac{\partial q^b}{\partial b'} db' + \frac{\partial q^b}{\partial f'} df'\right) + \vartheta^f f \left(\frac{\partial q^f}{\partial b'} db' + \frac{\partial q^f}{\partial f'} df'\right)}_{\text{gain value of legacy debt}}$$

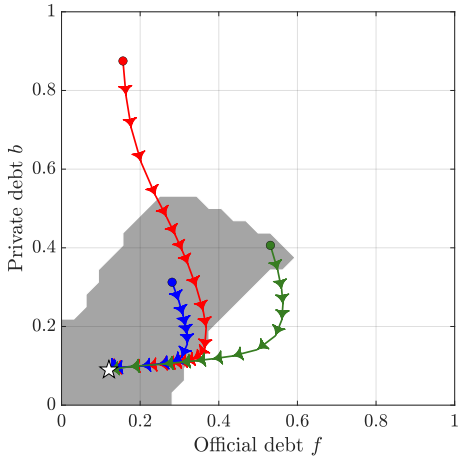
- Consider $V_{b'} db' + V_{f'} df' > 0$: it requires at least one type of debt to decrease
- Without legacy debt ($b = f = 0$) or uncertainty, $H_{b'} db' + H_{f'} df' < 0$ not Pareto improvement
- Sufficient gains from increase value of legacy debts necessary and/or uncertainty

Voluntary Swaps

Indifference curves



Dynamics



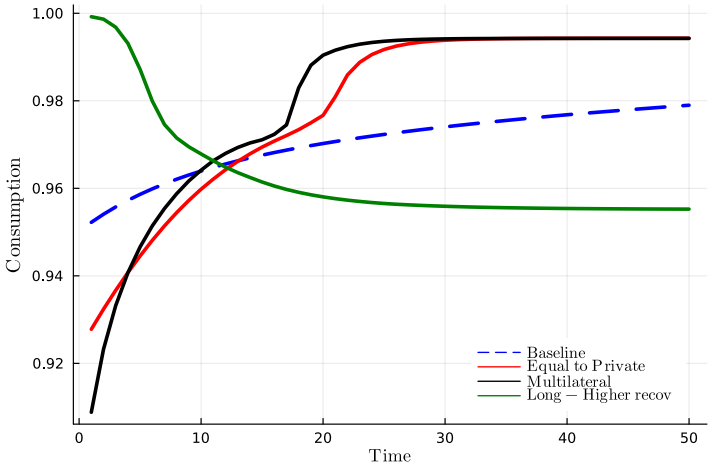
- Large set of state space with feasible swaps and 5% in limiting distribution
- A to B, 44% increase in H. A to C, 1.3% CE welfare gains.

Counterfactual Official Debt

	Baseline	Official Debt			
		Shorter	Lower recov.	Shorter + Higher recov. (Multilateral)	Longer + Higher recov.
Official debt	21	16	15	16	97
Private debt	13	13	13	11	16
Partial default	28	24	24	21	55
Consumption std. dev.	0.92	0.93	0.93	0.95	0.81
Welfare CE (%)					
No debts	0.00	-0.002	-0.02	0.13	0.04
Mean debts	0.00	-0.03	0.004	0.006	0.13
High debts	0.00	-0.07	0.06	-0.41	0.17

- Shorter duration and lower recovery reduce official debt capacity and increase consumption volatility
- Short is worse for welfare, lower recovery can been good with high enough debt
- Multilateral liquidity facilities (IMF) (2 year, 80% recovery) bad for welfare for indebted

Counterfactual Official Debt: Sources of Welfare



- Welfare comparisons reflect consumption dynamics in deleveraging episodes

Conclusion

- Official loans support economies during sovereign defaults
- With partial default, longer official debt gives greater debt capacity
- Model rationalizes the rising official debt during defaults
- Room for swaps of private for official during defaults (multilateral involvement make sense)