Official Sovereign Debt*

Cristina Arellano

Federal Reserve Bank of Minneapolis
and University of Minnesota

Leonardo Barreto

University of Minnesota and
Federal Reserve Bank of Minneapolis

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Abstract

This paper studies official sovereign debt empirically and theoretically. Official sovereign debt is more than half of the total sovereign debt in emerging markets and tends to flow in during default episodes. We develop a model with official and private debt where the sovereign can partially default on each of its debts. A fraction of the defaulted debt accumulates during a default episode, which resolves when the sovereign pays back its accrued obligations. Official debt is longer-term and more concessional during defaults than private debt, and the prices of all debts compensate lenders for default losses. The contractual differences across debts allow our model to rationalize the stylized facts of emerging markets. Counterfactual analysis suggests that official debt is welfare improving and finds the feasibility of voluntary swaps that generate Pareto Improvements by exchanging one type of debt for another one. Our work rationalizes the involvement of official debt in the resolution of sovereign defaults.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Contact information: arellano.cristina@gmail.com; barre843@umn.edu

1 Introduction

A large portion of the sovereign debt in emerging markets is with official lenders including bilateral loans with other sovereign governments and loans with multilateral organizations. The recent empirical work of Horn, Reinhart, and Trebesch (2020) emphasizes the importance of official lending for many countries historically and its role in coping with adverse shocks. Theoretical work on sovereign debt, however, has mainly focused on debt and default problems in contracts with private creditors. This paper provides an integrated framework with official and private debts and explores the role of official debt in helping during episodes of financial distress.

We start by analyzing a panel dataset covering 50 years and 30 emerging markets that contains debt series and their decomposition across official and private creditors. We build on the accounting framework of Arellano, Mateos-Planas, and Ríos-Rull (2023) to classify partial defaults and default episodes. We find that official debt corresponds to more than half of the total external sovereign debt for these countries. We document that official debt tends to grow during default episodes and accounts for much of the dynamics of total debt during these events. Private debt, in contrast, remains pretty stable during these episodes. We also present evidence that official lending has been an important component in the resolution of default episodes, such as the Brady Plan of the early 1990s.

We then develop a sovereign default model with official and private debts. Our framework consists of a sovereign in a small open economy that faces a stochastic endowment stream and can choose to partially default selectively on its coupon payments of official or private debt. Official debt differs from private debt in that it is of longer duration and more concessional, as it calls for lower recoveries during defaults. When the sovereign partially defaults on part of the official or private debt, a fraction of that amount, which depends on a recovery factor parameter that is debt-specific, accumulates as arrears and is added to the total debt next period. Partial default is costly because it induces resource costs that depend on the intensity of the default, yet it is a flexible policy as the government decides on the intensity of default, separately for each type of debt, as well as the number of periods with default. The sovereign raises funds by borrowing from official and private lenders at interest rates that compensate for potential default losses. Borrowing with these two assets is always possible, even during default episodes.

An important aspect of our framework is that partial default on each type of debt is a period-by-period decision by the sovereign and borrowing is permitted during default.[1] We show that partial

1. These features are different from traditional sovereign default theory with multiple debts, as in Arellano and Rama-

default incentives are higher when the values of official and private debt are large as well as when bond prices on new borrowings are low. Bond prices matter for partial default because they encode the value of the accumulated debt in arrears; a low price lowers the value of those claims and increases the benefit to the sovereign for defaulting today because less will be repaid in the expected value.

The intensity and length of default episodes are endogenous and depend on the dynamics of debts during the episode. A highly indebted sovereign may choose to default on most of its debt, but will tend to deleverage over time and reach a state with less debt and no default. We find that during the deleveraging process, the sovereign tends to reduce private debt first by contracting consumption and temporarily issuing official debt, to only later reduce also official debt. The speed and portfolio during the deleveraging process in turn depend on the bond price functions; tight bond price functions incentivize faster deleveraging. Bond prices functions of official debt tend to be more favorable, which is why official debt is used more heavily. Importantly these dynamics resemble the data in emerging markets during default episodes.

A main theoretical finding is that in our model official debt gives greater debt capacity to the sovereign relative to private debt. We characterize this result theoretically in a simplified model economy with no shocks, linear utility, and zero recovery factors upon a partial default. We show that borrowing with official debt expands the budget set of the sovereign more than borrowing with private debt. This is because official debt is a long-term duration asset and default is partial and a period-by-period decision. Official contracts can effectively constrain future governments from borrowing, as any pledgeable resources in the future, namely the default costs, can be used to increase the commitment to a long-term official contract that includes coupon payments during these future periods. In contrast, only the one-period ahead default cost provides commitment for private short-term contracts. Future periods' default costs are irrelevant to the commitment of the short-term private contract because, after default on short-term debt, the sovereign can borrow fresh loans again with new private lenders. We also relate to traditional sovereign default theory and show that the debt capacity of both assets would be the same under the common assumption of full default, no borrowing during defaults, and permanent costs.

We perform a quantitative evaluation of the model and map it to emerging market data. We use unconditional moments in the parameterization of the model and show that the model can reproduce salient patterns during default episodes. We target first and second moments of official and private

narayanan (2012), Hatchondo, Martinez, and Sosa-Padilla (2016) and Aguiar et al. (2019), where default is assumed to be a long-lasting decision that eliminates current and future obligations of both types of debts and precludes any borrowing.

debt and debt service and partial default. Our moment-matching process recovers parameters for the duration of debts, the recovery factors, and default costs. The exercise results in official debt is of longer duration and more concessional. Our baseline model reproduces an economy with debt ratios as in the data; official debt is about 2/3 of the total debt. Moreover, we show that as in the data, in the model debt grows during defaults, more so for official debt. The magnitudes of these increases are very similar to those in the data, which provides an important validation of the model. We also show that in the model when the sovereign exits default episodes, it first reduces private debt consistent with the dynamics in the data.

We use the baseline model to perform counterfactuals. We first evaluate the feasibility of voluntary swaps across official and private debt that generate Pareto improvements. We find a sizable region of the state space where swaps are feasible, especially when private debt and default risk are high. Swaps tend to exchange private debt for official debt, highlighting that official debt can play an important role in resolving financial distress events. Second, we study the design of official contracts by comparing our baseline with economies that feature official debt that is shorter-term and less concessional. These experiments are motivated by various liquidity programs from multilateral organizations. We find limited welfare benefits from these programs, and find instead that the best design consists of long-duration bonds.

Literature Review. Our work contributes to the literature studying official lending. Horn, Reinhart, and Trebesch (2020) documents how extensive official lending has been historically for many countries around the world. They make a compelling case that an important role of official debt is coping with the economic consequences of disasters, including natural and financial adverse shocks. Schlegl, Trebesch, and Wright (2019) studies the seniority of official and private debt. Using measures of debt in arrears and haircuts from default episodes, they argue that bilateral official lenders are more junior than private debt creditors. The rising role of official Chinese lending to emerging markets since the early 2000s, as documented by Horn, Reinhart, and Trebesch (2021), has also sparked some work focusing on official loans from China.[2] Our work complements these empirical findings, by using a comprehensive dataset on official and private debt and arrears. Our work documents a novel property, namely that official debt is used more heavily during default episodes.

Some theoretical work on official debt has studied multilateral lending and taken the approach of

2. See also Clayton et al., forthcoming on the role of China's bond markets more broadly in the international financial markets.

making these loans not defaultable. Boz (2011) and Kirsch and Rühmkorf (2017) enrich a sovereign default model with this type of multilateral lending and show that it can be useful at providing insurance. Our work on official lending also finds that this lending is useful in times of crises, but unlike previous work and consistent with the evidence allows for this debt to also be defaultable.

A main property of official lending is its long maturity. As such, our work contributes to the large literature that has analyzed the interactions between default risk and the maturity of sovereign debt. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that although the inclusion of long-term bonds improves the quantitative performance of sovereign debt models, it also tends to decrease welfare because of the so-called "debt dilution problem." Consistent with this work, Arellano and Ramanarayanan (2012) argue that short-term debt is better at providing incentives to repay, although long-term debt can provide hedging benefits. Aguiar et al. (2019) shows that these incentive benefits imply that the sovereign should actively manage only short-term debt while remaining passive in long-term bond markets, when income fluctuations are not a concern. Hatchondo, Martinez, and Sosa-Padilla (2016) studies the impact of debt dilution and shows that eliminating dilution can reduce default risk and increase welfare substantially.[3]

We re-consider these interactions in the framework of rich default episodes and partial default of Arellano, Mateos-Planas, and Ríos-Rull (2023). We find that studying maturity in this framework overturns the conventional result that short-term debt provides more repayment incentives and is preferred. In our framework, official long-term debt welfare dominates short-term debt because it carries higher debt capacity. These results are relevant because the framework of partial default can better resemble emerging market data.

Our findings of the possibility of Pareto improvements through voluntary debt swaps relate to those in Hatchondo, Martinez, and Padilla (2014). They study swaps in a framework of a single long-term defaultable bond and show that swaps can arise in equilibrium because the sovereign may borrow loans that reduce the value of the legacy debt. In our model with two types of debts, these forces are compounded as new private or official loans may dilute the value of both, private and official legacy debts.[4] Moreover, our emphasis on official lenders can alleviate the concerns in Bulow, Rogoff, and Dornbusch (1988), about the country being unable to obtain any benefits from buybacks, as official lenders can help coordinate the distribution of gains. We also show that the type of swaps we consider,

3. Other important studies on maturity and sovereign default risk include Sánchez, Sapriza, and Yurdagul (2018), Dovis (2019), Bocola and Dovis (2019), Mihalache (2020), and Bigio, Nuño, and Passadore (2023).

4. Also related is Aguiar and Amador (2024) which studies the feasibility of swaps across debts of different maturities in the presence of self-fulfilling runs and finds additional forces that can lead to feasible swaps.

which reduce private debt obligations with an increase in official debt, have empirical relevance in salient examples like the Brady Plan, which we discuss in the empirical section.

2 Empirical Properties of Official and Private Debt

In this section, we document some properties of official and private lending and sovereign partial defaults using 50 years of data from emerging markets. We will extend the analysis in Arellano, Mateos-Planas, and Ríos-Rull (2023) (thereafter AMR) by focusing on differential patterns of official and private debt using their accounting framework to organize the data.

2.1 Accounting

To fix ideas for our work, we will revisit the accounting framework of AMR and apply it to our case of analyzing disaggregated private and official debt. We will apply this accounting framework to the data from emerging markets and to the simulated data from the model.

Flow Financial Variables. Each period, the sovereign owes official lenders an amount $\tilde{f}_t$ and private lenders an amount $\tilde{b}_t$, which are the sum of all the coupons from past issuances due at t. As we will see later, these amounts include not only the promised coupons at t from newly issued bonds in previous periods but also the current obligations that result from past partial defaults. We consider a flexible partial default policy that is applied to the payment dues, given by d_t^f and d_t^b for official and private debt respectively. Defaults $\{d_t^f, d_t^b\}$ imply that the sovereign pays in period t the amount $(1 - d_t^f)\tilde{f}_t$ to official lenders and $(1 - d_t^b)\tilde{b}_t$ to private lenders, and does not pay $d_t^f \tilde{f}_t$ and $d_t^b \tilde{b}_t$. Given the default policies, debt service for official and private debt are $(1-d_t^f)\tilde{f}_t$ and $(1-d_t^b)\tilde{b}_t$, debt service for total debt is the sum of these two debt services. Partial default for each type of debt is defined as the fraction of the debt due defaulted on and therefore equals d_t^f and d_t^b for official and private debt. Partial default for total debt is defined as $d_t = (d_t^b \tilde{b}_t + d_t^f \tilde{f}_t)/(\tilde{b}_t + \tilde{f}_t)$.

Long-Term Bonds with Partial Default. We map the data into a tractable structure for long-term debt contracts that consist of perpetuity bonds with coupon payments that decay, as in Hatchondo and Martinez (2009). We allow for different durations for each type of debt by considering different decay parameters for the bonds, ϑ^f and ϑ^b for official and private debts respectively. For each type of debt $a \in \{f, d\}$, a borrowing contract specifies a price q_t^a and a value ℓ_t^a such that the sovereign receives

$q_t^a \ell_t^a$ units in period t and promises to pay, conditional on not defaulting, $R^a(\vartheta^a)^{n-1}\ell_t^a$ units in every future period $t+n$ for $n = 1, 2, \ldots, \infty$. The coupon rate R^a is a normalization that has no bearing on the analysis; it only changes the units of the debt due, such that $\tilde{a}_t = R^a a_t$, and will be set to $R^a = R - \vartheta^a$, where R is the gross risk-free rate. These settings mean that the default-free discount price for each contract is 1. These contracts are tractable because they encode a rich structure of debt issuances into a single state variable for each type of debt through their laws of motion. A sovereign that in period t pays in full its debts due $R^a a_t$ and borrows ℓ_t^a will have in period $t+1$ states equal to $a_{t+1} = \vartheta^a a_t + \ell_t^f$ for $a \in \{f, b\}$, namely official and private debt respectively. These states include the coupons from the legacy debt and the new borrowing.

We assume that partial default of intensity d_t^a for each type of debt a, reduces the debt service and can trigger defaults on all future coupons, encoded in the legacy debt $\vartheta^a a_t$, of intensity $\mu^a d_t^a$. We interpret the parameter μ^a as arising from default acceleration clauses, which are common in bonds, and also to reflect that many restructurings include bonds with streams of payoffs due in the future.[5] As in AMR, we assume that the defaulted coupons and defaults on legacy debt, namely $R^a d_t^a a_t + \vartheta^a \mu^a d_t^a a_t$, result in new obligations, $\kappa^a d_t^a a_t$, that are due in the future. The factor κ^a is a parameter that captures the empirical observation that during default episodes, sovereigns accumulate their defaulted debt and, in some cases, restructure their obligations with their creditors. All else equal, contracts with lower κ^a are more concessional: defaults on those contracts result in higher discharge of debt.

The law of motion of the states, therefore, incorporates the legacy debt, the accumulation of defaulted coupons, and new borrowing:

$$a_{t+1} = \vartheta^a(1 - \mu^a d_t^a)a_t + \kappa^a \, d_t^a \, a_t + \ell_t^a \tag{1}$$

Note that a partial default $d_t^a > 0$ does not necessarily reduce debt a_{t+1} relative to a_t. Debt can actually increase when the recovery factor is sufficiently high because the defaulted coupons are accumulated with interest. Also, debt can increase if borrowing ℓ_t^a is positive. As we document below, we find differential patterns of official and private debt during partial defaults.

5. Acceleration clauses are a feature in sovereign debt contracts that entitle creditors to accelerate unmatured principal following a default event. They are common in bonds issued under New York law and typically require a minority volte of at least 25% of the value. Accelerations can also be revoked (de-accelerated) by majority bondholders. See Das, Papaioannou, and Trebesch (2012) and Stefanescu (2016).

Debt, Duration, Spreads, and Default Episodes. We measure the level of official and private debts at t as the present value of the contractual payments due and the duration of these debts as the corresponding "Macaulay duration" with flows discounted at the risk-free gross interest rate R. Our bond structure implies that a sovereign with end-of-the-period state a_{t+1} has a level of debt equal to a_{t+1}, with associated duration of $\frac{R}{R-\vartheta^a}$.

In practice, because of default risk, the market value of the debt is different from the level of debt defined above. As is standard, we can use the market value of the debt and the streams of contractual payments to define the yield-to-maturity, which is the constant discount rate that equates these two. The sovereign spread s_t is the difference between the yield-to-maturity and the risk-free rate. For our perpetuity contracts, the market value of debt of class a is $q_t^a(R-\vartheta^a)a_{t+1}$ as future defaults are applied uniformly across all these securities of class a. The sovereign spread is inversely related to q_t and equals $s_t = (R-\vartheta^a)\left(\frac{1}{q_t^a}-1\right)$.

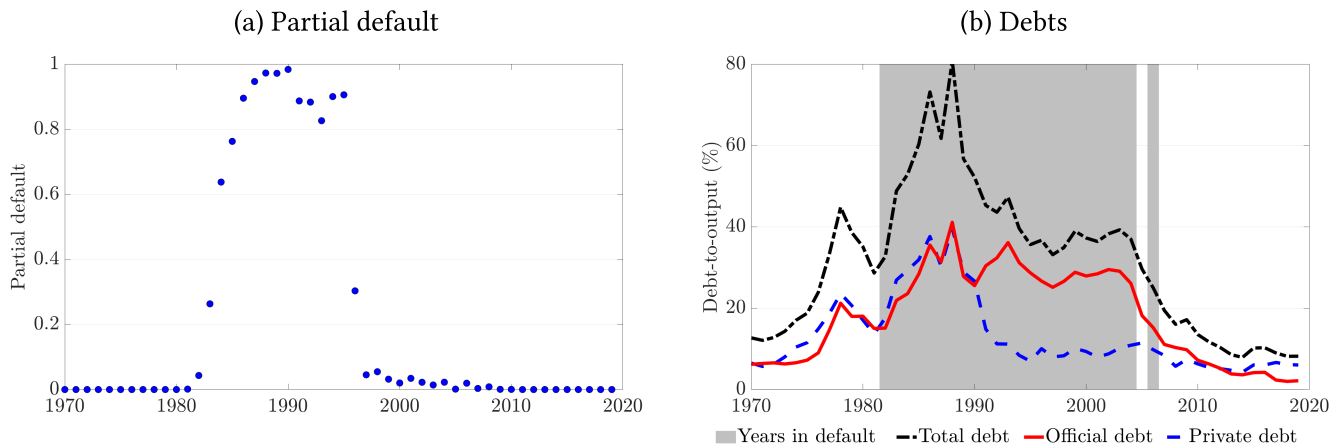
We flag a default episode as a sequence of periods with consecutive positive partial defaults and define its length by the number of such periods. An episode of length $N+1$, which starts in period t and ends in period $t+N$, has $d_{t+j} > 0$ for $j = 0, 1, \ldots, N$ and $d_{t-1} = d_{t+N+1} = 0$. The sequences of official and private debt level, debt service, as well as the sequence of partial default for official, private, and total, in the default episode, are given by $\{a_{t+j}, (1-d_{t+j})(R-\vartheta^a)a_{t+j}, d^a_{t+j}, d_{t+j}\}$ for $a \in \{f, b\}$ and $j = 0, 1, \ldots, N$.

2.2 Empirical Findings

We use the debt statistics from the World Development Indicators (WDI), International Debt Statistics (IDS), and the Debtor Reporting System, all from the World Bank, to empirically measure the variables of interest in our accounting framework at an annual frequency. From these data, we use the debt obligations for the government, defined as public and publicly guaranteed (PPG), for both flow and stock variables. We focus on the total debt obligations, as well as the decomposition across these obligations between official and private credit. Debt obligations with private creditors include debt in the form of bonds and loans, and trade credit, and debt with official creditors includes loans with bilateral governments and multinational organizations. We also collect data on Gross Domestic Product in constant dollars which we log and linearly detrend. We also use government EMBI+ spreads from the Global Financial Database. The dataset is annual and corresponds to a panel of 30 emerging countries from 1970 to 2019.

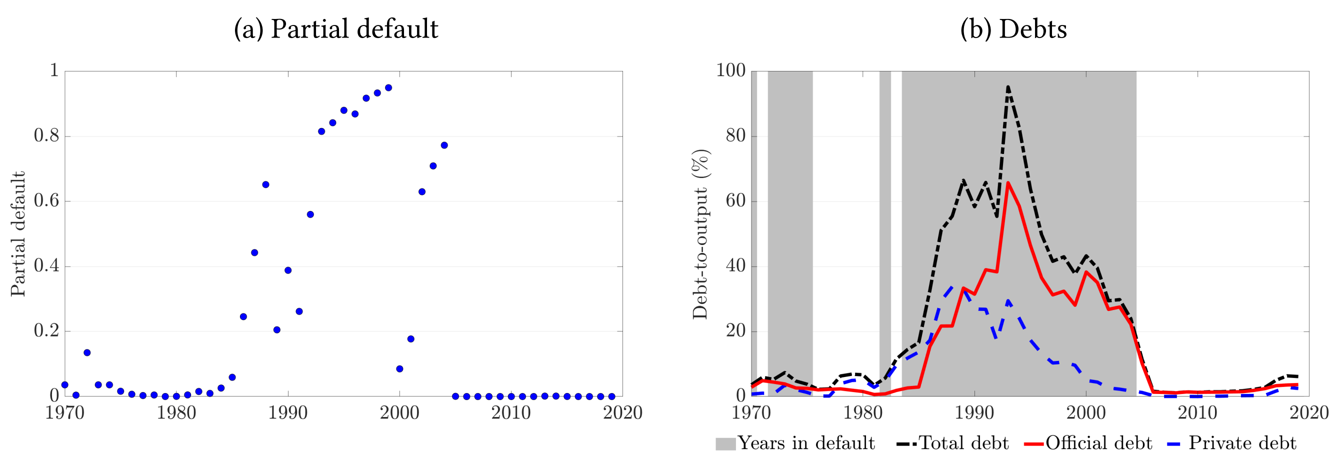
2.2.1 Case Studies

Figure 1: Peru: Partial Default, Official, Private, and Total Debt



Notes: Partial default is the ratio of total debt due in arrears over the sum to total debt service and debt in arrears due. Debt is external debt that is public and publicly guaranteed. Total debt corresponds to the total external, official is the sum of debt from bilateral and multilateral creditors, and private is the sum of debt for bonds and loans from private creditors, and trade credit. Data is from the World Bank.

Figure 2: Nigeria: Partial Default, Official, Private, and Total Debt



Notes: See notes of Figure 1

We start by describing the times series properties of partial default and total, official, and private government debt for two countries, Peru and Nigeria. These countries feature time series patterns that are similar to the average patterns across countries. Figure 1 plots the times series for Peru; the left panel in the figure has the time series for partial default, and the right panel contains the time series for the debts. The left panel shows that Peru had a long default event, with partial default increasing from the early 1980s from 0 to about 1 in 1990. Partial default fell after that but remained positive well into the early 2000s. The right panel plots the three debt series, the black dotted line corresponds to the total debt to output, the red solid line is the official debt to output and the blue dashed line is the private debt to output. The sum of official debt and private debt is the total debt. The figure also contains shaded bars, that correspond to the periods of positive partial default. Before the default episode started, the level of official and private debt in Peru was similar and equal to about 15% of output. When the default episode starts, both debts grow and reach close to 40% in the late 1980s. The official debt remains elevated until the end of the episode at about 30% of output. The private debt in contrast falls during the episode and remains at about 10% of output for much of the latter part of the episode. The total debt at the end of the default episode is similar to before the beginning of the episode, but this end level is largely composed of official debt, in contrast to the beginning, where the shares of official and private were very similar. After the episode, official debt falls as well.

Figure 2 plots the time series for Nigeria. The structure of the figure is the same as for Peru. Nigeria experienced four default episodes according to our accounting. We will focus on the long episode that starts in the mid-1980s and runs through the mid-2000s. The left panel shows that partial default starts small and increases during the episode reaching its peak in the mid-1990s. Early 2000s partial default is minor but in the mid-2000s it increases again. The right panel shows that the patterns of debts in Nigeria were similar to those in Peru with some differences. Right before the default episode starts, Nigeria had mainly private government debt, about 15% of output. The beginning of the default episode features a rise in both official and private debt. By the late 1980s, private debt stops rising but official debt rises to about 60% of output. At the end of the default episode, Nigerian debt is mainly official, at about 25% of output. After the episode, official debt falls too.

These examples illustrate that the dynamics of official debt are crucial for understanding the evolution of debt during default episodes. Official debt is a major source of financing during default episodes, and in some cases substitutes the use of private debt. Moreover, the increase in total debt is driven mainly by official debt and private debt tends to fall earlier in the episode relative to official debt.

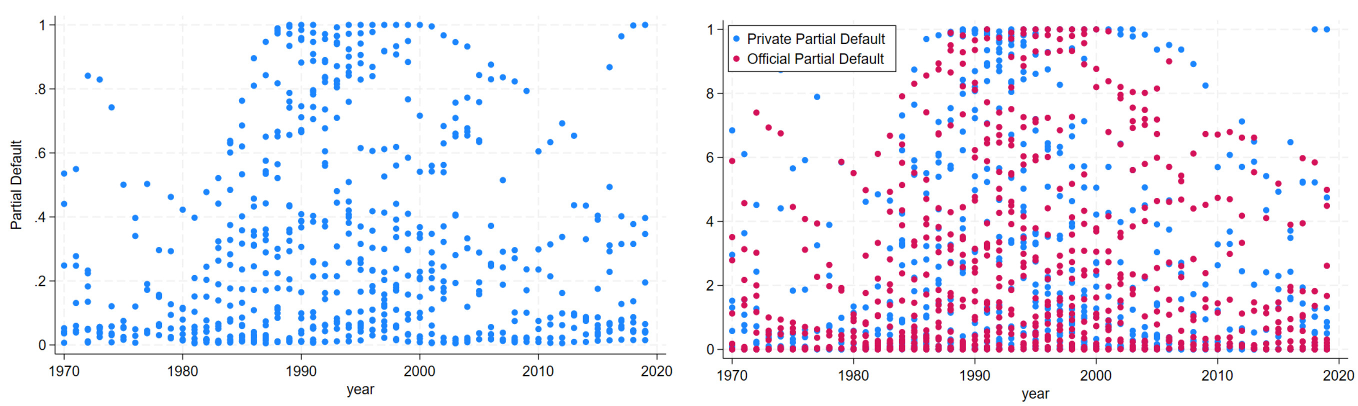
2.2.2 Descriptive Statistics

We now describe the panel data and start with the time series for partial default, for total, official, and private debt for the panel of countries. Panel (a) in Figure 3 plots the time series of partial default (conditional on positive) d_t and panel (b) plots partial default for official and private debts, d_t^b, d_t^f, for all the countries. The plots are conditional on positive partial default, which occurs with a frequency of 40%. There is a wide dispersion in partial defaults and partial defaults are very correlated. Moreover, during the 1980s and 1990s many countries partially defaulted on official and private debts.

We summarize these series in Table 1. Partial default, conditional on positive, is on average 32%, 31%, and 35%, for total, official, and private debt respectively. Partial default is highly volatile, with an average volatility across countries of 24%, 22%, and 27% for total, official, and private. All the partial default series are correlated. The bottom panel of the table reports that the mean correlation across countries between partial defaults on official and private debt is 62%, and the correlation between official and private debt is 42%.

Figure 3: Partial Default for Total, Official, and Private Debt

(a) Total Debt (b) Official and Private Debt



Notes: Partial default across countries and time. Panel (a) contains partial default based on total debt conditional on positive default. Panel (b) reports partial default separately for official and private debt. See the notes of Table 1 for more detailed descriptions of the series.

Table 1 also reports the first and second moments for debt levels and debt service for total, official, and private, all relative to output. The mean total debt to output in the panel data is 33%. The share of official debt is 61% and the share of private debt is 39%; official debt to output is 20% on average and private debt to output is 13%. Total debt service is 3.5% of output, about 50% of this debt service is paid to official lenders and half of it to private lenders. An interesting feature of this data is that although official debt is about 50% higher than private debt, the debt service of official debt is smaller than that for private debt. As we explore further in the calibration of our quantitative model, a higher level of debt relative to the coupon payments is consistent with longer duration debt.

The second column in Table 1 reports the standard deviations of the debts. The standard deviations reported are the average ones across the countries in the sample. The volatility of the debt is high, with comparable coefficients of variation across official and private debt. The volatility of debt service is 2%

and about half of that for official and private. Official and private debt also tend to move together, with a correlation of 42%, which makes them very correlated with total debt (88% and 73% respectively).

Table 1: Partial Defaults and Debts

		Mean	Std. dev.
Partial default | >0			
Total		32	24
Official		31	22
Private		35	27
Debt to output			
Total		33	18
Official		20	12
Private		13	8
Debt service to output			
Total		3.5	2.0
Official		1.6	1.0
Private		1.9	1.6
Corr. (official partial default, private partial default)	62		
Corr. (official debt, private debt)	42		

Notes: Data is from the World Bank Databases and measured as public and publicly guaranteed (PPG). Total corresponds to the total external PPG series. Official is the sum of debt from bilateral and multilateral. Private is the sum of debt for bonds, loans, and trade credit with private creditors. Partial default is the ratio of debt in arrears over the sum of debt service and debt in arrears for total, official, and private. The standard deviations are means across countries of the statistics using country time series data. All variables are expressed in %.

Official Debt and Default. We now assess how various variables of interest vary with partial default. Table 2 reports means across states with no partial default and positive partial default, based on total debt. Debt to output is about 20% higher when partial default is positive (44% vs 24%). The majority of this increase is due to an increase in official debt. Official debt to output increases by 16% while private debt to output increases only by 4%. Interestingly, we also find debt service to output increases on average with partial default—although the government is not paying all of its debt due, the higher debt implies that the government is paying more for servicing the non-defaulted portion of the coupons. The increase is more for official debt because of the additional inflows of this type of debt during these times.

The table also illustrates that periods with positive partial default are associated with higher sovereign spreads and lower output; spreads are about 7% higher with partial default and output is 5% lower.[6]

We now further decompose the patterns of debt across finer partial default bins. In Figure 4, we report official and private debts across four bins for partial default. The bin *No default* contains the observations with no partial default. The bin *Small* contains observations with positive partial default but below the 25 percentile. The bin *Medium* contains observations with partial default between the 25 and 75 percentile. The bin *Large* contains the observations with partial default above the 75 percentile. The bars are the means of official and private debt to output across bins; the red bars correspond to official debt and the blue bars correspond to private debt. The figure illustrates that both debts increase with partial default and that the increase is sharper for official debt. Official debt is about 25 percentage points higher when partial default is in the top quartile (bin 4) relative to when partial default is zero (bin 0). Private debt in contrast is about 8% higher when partial default is in the top quartile relative to when it is zero.

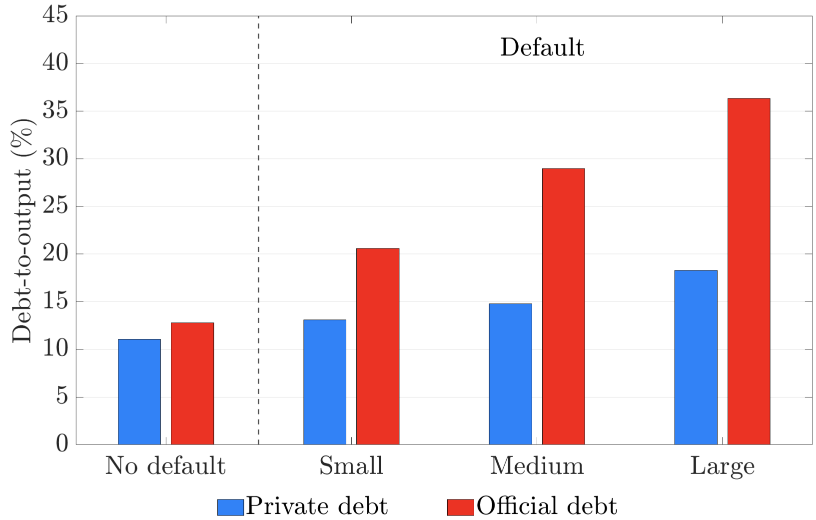
Table 2: Default Flag: Total, Official, and Private Debt

	No default	Partial default > 0
Debt to output		
Total	24	44
Official	13	29
Private	11	15
Debt service to output		
Total	3.0	4.1
Official	1.2	2.1
Private	1.8	2.0
Spreads	4	11
Output	2	-3

Notes: The statistics are means of the variables in the first column after partitioning the panel data set across two bins based on partial default based on total debt. All variables are expressed in %. The "No default" bin has all the observations with zero partial default; the "Partial default > 0" bin has the observations with positive partial default. Output is logged and detrended using the Hodrick-Prescott filter with a multiplier of 6.25. We measure private spreads with the spread series of the Emerging Market Bond Index (EMBI+) from the Global Financial Database for each of the countries in our sample. See also the notes of Table 1 for additional details.

6. In the Appendix A, we show that these patterns are not driven by fluctuations in output. These conditional statistics for debts are very similar when constructed relative to trend output. We also show that the patterns of multilateral and bilateral debt are similar to that of official debt.

Figure 4: Official and Private Debt across Partial Default Bins



We now study the properties of default episodes for the 30 emerging markets, by analyzing dynamics within default episodes for the variables of interest. Using our accounting framework we measure 62 default episodes in our dataset. The average length of the default episode is 10 years, but many defaults are shorter, about 35% of default episodes last less than or equal to 2 years.

Table 3 reports the patterns of partial default and debt to output for total debt, official debt, and private debt during default episodes. We also report the patterns of spreads and output. We report average statistics for these variables for the period before the start of the default episode, labeled *Before*, the first period of the default episode, which we label *Beginning*, the middle of the default episode, which we label *Middle*, and the period after the end of the episode, when partial default returns to zero, which we label *After*.[7] As in AMR, we find that partial default on total debt and total debt to output feature hump shape patterns within episodes. The patterns of partial default on official and private debt are very similar to that on total debt within the episode, illustrating the strong co-movement among these variables discussed above. In contrast, the dynamics across debt types feature some distinct patterns and implications. First, the decomposition shows that the dynamics of total debt are mainly driven by the dynamics of official debt. Official debt to output increases by about 7% during the episode while private debt to output increases only by 1%. This evidence reinforces the point that official debt flows in during sovereign defaults. A second point is the deleveraging process prior to the end of the default episode occurs by reducing both types of debt. In fact, arguably private debt is reduced more aggressively as

7. We define the middle of the episode as the total length of the episode divided by 2, rounded to the nearest integer.

Table 3: Dynamics during Default Episodes

	Before	Beginning	Middle	After
Partial default	0	14	26	0
Official	0	8	22	0
Private	0	14	23	0
Debt to output (in %)				
Total	33	35	40	33
Official	17	18	24	19
Private	16	17	17	14
Spreads	11	21	16	5
Output	0	-2	-5	-3

Notes: The dynamics of the default episode are averages across the 62 episodes in our sample for the variables of interest. *Before* is the period before the start of the episode; *Beginning* is the first period of the episode; *Middle* is the midpoint of the episode; *After* is the period when partial default returns to zero. Debt is reported relative to output; output is logged and linearly detrended and reported relative to the level before the episode. See notes in Table 1 for the definitions of the variables.

it reaches a level below that observed before the default episode, while official debt continues to be elevated. The table also shows that spreads feature a hump-shaped pattern while output features a U -pattern. In Appendix A, we also provide robustness results. We show that the patterns of official and private debt across defaults are similar when defining debt ratios with trend output. We also find that within official debt, the both bilateral and multilateral debts increase with default.

Restructurings under the Brady Plan. Many emerging markets in our sample experienced sizable debt crises during the 1980s, which were resolved under the Brady Plan. The Brady Plan provides an interesting example of official creditor involvement for the resolution of defaults. Here we review some of these details to shed light on how official debt increases during default episodes.

By late 1980s, many countries had substantial private debts with commercial banks and much of it was in default. The debt crises had been evolving for most of the decade and included multiple rounds of unsuccessful restructurings, with countries defaulting on the restructured debt numerous times. This multi-country decade-long crisis was resolved under the Brady Plan, named after U.S. Treasury Secretary Nicholas Brady. As described by Truman (2023), the program involved not only the emerging market sovereigns and their private lenders, but a collection of official lenders, notably the International

Monetary Fund, the World Bank, lenders form The Paris Club, and the U.S. government.

The Brady plan consisted of a comprehensive restructuring program. The program was implemented first in 1989 by Mexico, followed by over 17 other sovereigns, and concluded in the mid-1990s. An important element of this process was the cash payments through market-based buybacks offered as part of debt restructuring to private lenders. As described by Zettelmeyer, Savastano, and Lui (2021), the IMF and the World Bank provided funds to facilitate these buybacks. In particular, the Debt and Debt-Service Operations (DDSRO) plan and the Debt Reduction facilities from these organizations provided important financing. This paper documents that 11 countries obtained IMF funding which was used to buy back debts and to purchase the collateral needed for the bonds. The IMF funds were effectively loans, which increased the indebtedness of the countries. These loans required changes in IMF's policies to allow lending to countries with debt in arrears.

A second important element of the Brady Plan, was that the new bonds used in the exchange, were partially collateralized. In most cases, the principal of the bonds was collateralized by 30-year U.S. treasury bonds and the escrow agent holding the collateral was the Federal Reserve. As argued by Truman (2023), this involvement and support of the U.S. government helped give the program increased credibility and was an important component in the resolution of the debt crises.

Summary. We conclude this section by summarizing our findings from our emerging market data. First, we document that official debt is large in emerging markets, and represents more than half of the external debt for governments. Second, we document that official and private debt grows with partial default, but the increase is sharper for official debt. Third, we find that private debt returns to lower levels after default episodes while official remains elevated. Finally, we find evidence of official credit involment in the resolution of debt crises. In the next section, we develop a model with official and private sovereign debt and default to study theoretically the patterns of these debts during default episodes and rationalize these patterns.

3 The Model Economy

We consider an infinite horizon model of sovereign default with official and private debt. The sovereign has preferences over consumption

$$\mathbb{E}_t \sum \beta^t u(c_t)$$

where β is the discount factor and $u(c)$ is increasing and concave. The economy faces stochastic endowment z_t and can borrow from international official and private lenders. Official debt is denoted by f_t and private debt by b_t. Debt contracts are perpetuities with coupon payments equal to $R^f f_t$ and $R^b b_t$ and that decay at rate ϑ^f and ϑ^b, respectively for official and private debt.[8] The sovereign can selectively partially default on each type of debt; the partial default decision for official debt is given by $d_t^f \in [0,1]$ and that for private debt is $d_t^b \in [0,1]$. A partial default of intensity d_t^a for debt of type $a \in \{b, f\}$ means that the sovereign defaults on $d_t^a R^a a_t$ of the coupons due upon the partial default and also defaults on $\mu^a d_t^a$ of all future debt coupons for that type of debt. Partial defaults lower resources for absorption, such that output depends on the endowment as well as partial defaults, $y_t(z_t, d_t^f, d_t^b) \leq z_t$ when $d_t^f > 0$ or $d_t^b > 0$. Partial default reduces the payments on the official or private coupons, but a fraction of the defaulted coupons accumulate and are due in the future. The fraction of defaulted coupons that accumulate is κ^f for official debt and κ^b for private debt.

The sovereign borrows new official loans ℓ_t^f at price q_t^f and private loans ℓ_t^b at price q_t^b to support consumption and pay off the existing debt due. The budget constraint for the sovereign is

$$c_t = y_t - (1 - d_t^f) R^f f_t - (1 - d_t^b) R^b b_t + q_t^f \ell_t^f + q_t^b \ell_t^b$$

Consumption equals output net of the non-defaulted coupon payments of official and private debt and new loans. The structure of our perpetuity contracts and accumulation of coupons with partial default gives rise to the following laws of motion for each type of debt due

$$a_{t+1} = \vartheta^a (1 - \mu^a d_t^a) a_t + \ell_t^a + \kappa^a d_t^a a_t \qquad \text{for } a = f \text{ or } b \tag{2}$$

The laws of motion incorporate the coupons from the legacy debt that are not defaulted on $\vartheta^a a_t (1 - \mu^a d_t^a)$, the new issuances ℓ_t^a, and the accumulation of the defaulted coupons $\kappa^a d_t^a a_t$. Note that $\kappa^a / (R^a + \mu^a \vartheta^a)$ can be interpreted as the recovery rate for a one-period default. The present value of the recovered debt is $\kappa^a d_t^a a_t$ and the present value of the defaulted debt is $(R^a + \mu^a \vartheta^a) d_t^a a_t$; the ratio of these two is therefore the recovery rate.

The prices for official and private loans are schedules that compensate lenders for the losses from default. As we will see below, this means that these schedules depend on the state of the following

8. The coupon rates R^f and R^b are constant rates that scale up the units of the debt payments. They will be set so that the default-free discount prices of the perpetuity contracts are equal to one, $R^a = (R - \vartheta^a)$ for $a \in \{f, b\}$, where R is the gross risk-free interest rate.

period $\{f_{t+1}, b_{t+1}\}$ and on the endowment z_t because it is useful to forecast the endowment the following period. These schedules are $q^f(f_{t+1}, b_{t+1}, z_t)$ and $q^b(f_{t+1}, b_{t+1}, z_t)$.

3.1 Recursive Problem for the Sovereign

The state variable for the sovereign includes the official and private debt and the endowment, $s = \{f, b, z\}$. The sovereign also takes as given the bond price functions for the two types of debt. Given these states and the bond price functions, the sovereign makes choices for partial defaults, official and private loans, and consumption to maximize its value

$$V(f, b, z) = \max_{c, d^f, d^b, \ell^f, \ell^b} u(c) + \beta \mathbb{E}\left[V(f', b', z')\right] \tag{3}$$

subject to the budget constraint

$$c = y(z, d^f, d^b) - (1 - d^f)R^f f - (1 - d^b)R^b b + q^f(f', b', z)\ell^f + q^b(f', b', z)\ell^b, \tag{4}$$

the accumulation equations of official and private debt in (2), and the restriction that partial default on official and private is bounded, $0 \leq d^f \leq 1$ and $0 \leq d^b \leq 1$. This problem results in decision rules for consumption, partial defaults, official and private borrowing, denoted by $\mathbf{c}(f, b, z)$, $\mathbf{d^f}(f, b, z)$, $\mathbf{d^b}(f, b, z)$, $\boldsymbol{\ell}^{\mathbf{f}}(f, b, z)$, and $\boldsymbol{\ell}^{\mathbf{b}}(f, b, z)$. We can use the decision rules for partial default and borrowing to determine the decision rule for next period's debts $\mathbf{f}'(f, b, z)$ and $\mathbf{b}'(f, b, z)$ as dictated by the laws of motion.

3.2 Loan Contracts

International lenders are competitive, discount the future at rate R, and do not have any recourse other than that dictated by terms of the contracts. Bond prices are functions that depend on $\{f', b', z\}$ to compensate lenders for the expected loss of default which depends on these states. The bond price function for private and official loans, with $a \in \{b, f\}$, satisfy

$$q^a(f', b', z) = \frac{1}{R}\mathbb{E}\left[(1 - \mathbf{d^{a}}')R^a + (\vartheta^a(1 - \mu^a \mathbf{d^{a}}') + \mathbf{d^{a}}'\kappa^a)q^a(\mathbf{f}'', \mathbf{b}'', z')\right] \tag{5}$$

The expression for the bond price encodes the expected stream of payments per unit of the loan for the life of the perpetuity contract. The first term $(1 - \mathbf{d^{a}}')R^a$ is the expected payment of the first coupon of the bond in the period following the issuance and it takes into account the potential partial default.

The second encode that the perpetuity contract calls for the long-term promise to pay ϑ^a fraction of the coupon the following period net of the reduction from the partial default $(1-\mu^a \mathbf{d^a}')$. The third term $\mathbf{d^a}'\kappa^a$ takes into account that κ^a fraction of the defaulted coupons remain as future obligations. The future obligations arising from defaulted coupons and long-term promises contain default risk and a specific coupon structure, both of which are encoded in the continuation price $q^a(\mathbf{f}'', \mathbf{b}'', z')$. Importantly, this future bond price is evaluated at the equilibrium policy functions given a particular choice $\{f', b'\}$.

It is also useful for some of the counterfactuals we analyze below to define the values to official and private lenders to claims for debts given a state. We denote by $H^f(f, b, z)$ and $H^b(f, b, z)$ the values for lenders per unit of private and official debt respectively. Given a state $\{f, b, z\}$, the value for private and official lenders equal the expected payments and satisfy

$$H^a(f, b, z) = [(1 - \mathbf{d^a})R^a + (\vartheta^a(1 - \mu^a \mathbf{d^a}) + \mathbf{d^a}\kappa^a)q^a(\mathbf{f}', \mathbf{b}', z)] \qquad \text{for } a = f \text{ or } b \tag{6}$$

Given a level of debts (f, b), values are low if default is high, recovery net of acceleration $\kappa^a - \mu^a \vartheta^a$ is low, and continuation prices are low $q^a(\mathbf{f}', \mathbf{b}', z)$. These continuation prices reflect expected losses from future default and depend on the dynamics of debts.

3.3 Characterization of Partial Default

We now characterize the partial default decisions. Given the structure of our model, we can recast the sovereign problem in two stages. In the first stage, the partial default policies for official and private debt are determined given a state and any potential choices of future states $\{f', b'\}$. In the second stage, the sovereign makes its portfolio choices for official and private $\{f', b'\}$. For the first stage, in an interior optimum, the partial default policies are chosen to expand the budget set of the sovereign and satisfy the following conditions:

$$\begin{aligned} -y_{d^f}(z, d^f, d^b) &= f[R^f + q^f(f', b', z)(\vartheta^f \mu^f - \kappa^f)] \\ -y_{d^b}(z, d^f, d^b) &= b[R^b + q^b(f', b', z)(\vartheta^b \mu^b - \kappa^b)] \end{aligned} \tag{7}$$

The left-hand sides are the marginal costs of partial default for each type of debt in terms of output losses. The right-hand sides are the marginal benefits. Absent any accumulation of defaulted coupons, $\kappa^f = \kappa^b = 0$, nor default on legacy debt, $\mu^f = \mu^b = 0$, the marginal benefits are the expansion of resources from saving on the defaulted coupons for official debt $R^f f$ or private debt $R^b b$. With the accumulation of coupons and default on legacy debt, however, the marginal benefit is the present value from the change in debt obligations that results from the partial default evaluated at market prices.

Note that low bond prices increase the incentives to default because they reduce the value of the defaulted coupons. At an interior solution, the partial default policy equates the marginal costs and benefits. However, partial default is bounded, $0 \leq d^a \leq 1$ for $a \in \{f, b\}$. If the marginal costs strictly exceed the marginal benefits for any positive partial default for debt type a then $d^a = 0$; conversely if the marginal benefit exceeds the costs at $d^a = 1$, then default is full. Partial default incentives are higher also when the values of official and private debt are large and when the accumulation factors of the partial default $\{\kappa^f, \kappa^b\}$ are low. Given this partial default policy, the sovereign chooses official and private debt to maximize its value taking as given the bond price functions.

4 Model Characterization

In this section, we simplify the model and characterize a few key properties of official and private debt contracts. To that end, we assume that official debt are perpetuities with $\vartheta^f = 1$ and private debt are short-term contracts $\vartheta^b = 0$. We also normalize the coupon rates, such that default-free discount prices equal one for both debts, $R^f = r$ and $R^b = 1+r$, where $r = R-1$. For simplicity, we also consider the case of linear utility, no accumulation of defaulted coupons, no default on legacy debt and a fixed default cost with any positive default, such that $y_t = z_L$ if $d_t^f > 0$ or $d_t^b > 0$. We summarize these settings in the following assumption.

Assumption 1 (Simple Economy). *In the simple economy, $u(c) = c \geq 0$, $\vartheta^f = 1$, $\vartheta^b = 0$, $\kappa^f = \kappa^b = \mu^f = \mu^b = 0$, $R\beta < 1$, and initial debt is zero, $b_0 = f_0 = 0$. Absent default, productivity is constant $z_t = z$, and it falls to $z_t = z_L$ if $d_t^f > 0$ or $d_t^b > 0$. Default does not prevent new borrowing.*

Default and Budget Sets. As is standard in sovereign default models, default incentives shape the price schedules for debt, and these in turn determine the supply of loans. Here we use our simplified model to characterize default incentives and the price schedules. A main objective is to characterize how official and private debt differs in terms of default incentives.

Under assumption 1, the recursive problem for the government is the following

$$V(f,b) = \max_{c, d^f \in [0,1], d^b \in [0,1], \ell^f, \ell^b} c + \beta\, V(f', b')$$

subject to $c \geq 0$ and the budget constraints. With no default, $d^f = d^b = 0$, the budget is

$$c = z - rf - (1+r)b + q^f(f', b')(f' - f) + q^b(f', b')b'$$

Given that the default cost is fixed and independent of the intensity of the default, if the sovereign chooses to default, it fully defaults on the coupons of both debts, namely $d^f = d^b = 1$. The budget constraint with default is then

$$c = z_L + q^f(f', b')(f' - f) + q^b(f', b')b'$$

Importantly, default does not preclude market access to borrowing or paying future debt. Default is a period-by-period decision that is costly only because it reduces resources if coupons are not paid. Given the setup, the sets of contracts available for official and private debt, namely $q^f(f', b')f'$ and $q^b(f', b')b'$, do not depend on whether the sovereign defaults or repays the coupons. This means that default will be chosen if it expands the budget set and that the default policies are:

$$d^f = d^b = \begin{cases} 0 & \text{if} \quad rf + (1+r)b \leq z - z_L \\ 1 & \text{otherwise.} \end{cases} \tag{8}$$

The default policies map into price functions. To analyze the impact of default policies on bond prices and budget sets, it is useful to consider using one type of bonds at a time and the case when the economy starts with no debt in period 0, namely $b_0 = f_0 = 0$. Suppose first that the sovereign uses only private debt. Given default policies, the private loan that maximizes the budget that guarantees repayment is $b_{\max} = \frac{z - z_L}{1+r}$ and the associated price is $q^b = 1$.

Under Assumption 1, it is optimal for the sovereign to choose this loan in period 0. In period 1, the sovereign is committed to repay $(1+r)b_{\max}$ but it does not have any further commitments from the period 0 contract. However, in all future periods, it is optimal for the sovereign to exhaust its borrowing capacity. These policies imply that consumption paths with private debt contracts satisfy

$$c_0 = z + \frac{z - z_L}{1 + r} \qquad \text{for } t = 0$$
$$c_t = z_L + \frac{z - z_L}{1 + r} \qquad \text{for } t \geq 1$$

Consumption in period 0 is expanded by $\frac{z-z_L}{1+r}$ and in future periods consumption is independent of the period 0 private contract.

Suppose now that the sovereign only uses official debt. Unlike for private debt, the long-term nature of official debt implies its bond price function depends on all future default incentives and future borrowings. Consider a candidate official debt contract that gives the sovereign barely enough incentives to repay in the future. Given default decisions in (8), this contract has a coupon value that is equal to the cost of default, such that $rf = z - z_L$.

An official contract at $t = 0$, that incorporates a transfer to the sovereign of $f_1 = \frac{z-z_L}{r}$ and promises to pay $rf_t = z - z_L$ for $t \geq 1$, is the maximal contract that ensures repayment. It is optimal for the sovereign to choose this contract $f_{\max} = \frac{z-z_L}{r}$ with the associated price $q^f = 1$ and maximize its budget at $t = 0$. Consumption paths with official debt are therefore

$$c_0 = z + f_{\max} = z + \frac{z - z_L}{r} \qquad \text{for } t = 0$$
$$c_t = z - rf_{\max} = z_L \qquad \text{for } t \geq 1.$$

With the maximal official contract, consumption in period 0 is expanded by $\frac{z-z_L}{r}$, and is reduced in all future periods as the sovereign pledges future resources to servicing the official debt coupons.

The analysis comparing private and official loans gives our first result.

Lemma 1 (Official expands budget more). *Under Assumption 1,*

$$q^f(f'_{max}, b' = 0)f'_{max} = \frac{z - z_L}{r} > q^b(f' = 0, b'_{max})b'_{max} = \frac{z - z_L}{1 + r}.$$

This result arises because the official debt contract effectively constrains future governments from borrowing as the pledgeable resources, namely the default cost $z - z_L$, are already committed to the legacy official contract. The official contract can extract the present value of these resources, $(z - z_L)/r$. Private debt, in contrast, does not constrain future governments from borrowing, and therefore it can extract only the one period ahead pledgeable resources $(z - z_L)/(1 + r)$. This difference implies that private contracts cannot replicate the paths of consumption that are possible with official contracts. In models with sovereign default, debt maturity generally matters for allocations.[9] But the common result is the opposite: short-term debt can replicate long-term debt contracts, but not vice-versa. In our partial default model, however, the result is that only official debt which is longer-term, can replicate the allocations with only private debt, which is short-term. Next, we explore how our model's main differences relative to the standard model, namely borrowing during default and timing of default costs, give rise to these differences.

Assumption 2 (Permanent exclusion and output costs)**.** *In the simple economy, any positive default results in a permanent exclusion from financial markets and output costs.*

This is a standard assumption in the sovereign default literature in the tradition of Eaton and Gersovitz (1981), namely that default triggers a permanent cost in the form of exclusion from borrowing and output costs. Under assumption 2 the patterns of consumption change and therefore default decisions (8) are different. Consumption with default is equal to output net of the cost of default, $c = z_L$, while consumption during repayment depends on the policy functions for future official and private loans. Given the preference assumptions in (1), these policy functions are simple: the sovereign exhausts its borrowing capacity in every period. As above, let $b_{\max}$ and $f_{\max}$ be the maximum levels of private and official debt that prevent default. The following result shows that now both types feature the same debt limits.

Lemma 2. *Under Assumption 2, official and private loans expand equally the budget*

$$q^f(f'_{max}, b' = 0)f'_{max} = \frac{z - z_L}{r} = q^b(f' = 0, b'_{max})b'_{max} = \frac{z - z_L}{r}.$$

The standard assumption in the sovereign default literature of exclusion from financial markets after default is at odds with the empirical evidence documented above. During periods of default, sovereigns continue to participate in financial markets, and official loans in particular come in. We find interesting, however, that it is this assumption that leads to the possibility that short-term debt replicates long-term debt, in the presence of default. Under the more empirically relevant assumption in our baseline model, that default does not preclude borrowing, private shorter-term debt features more limited debt capacity than official longer-term debt.

9. See for example, Aguiar et al. (2019) and Arellano and Ramanarayanan (2012).

5 Quantitative Evaluation

This section presents the quantitative evaluation of our model. We first describe the parameterization of the model, which incorporates a moment-matching exercise that uses the panel data presented above. We describe the dynamic decision of the models and policy functions. We then compare the implications of the model for additional moments that characterize default episodes and find that it delivers patterns comparable to the data.

We then turn to the evaluation of the role of official lending for welfare through various counterfactuals. We first evaluate the scope of official lending in generating Pareto improvements by engineering swaps of the two different types of debt. We find that in regions of high private debt and low official debt there may be room for Pareto improving swaps. We finally study the design of official debt contracts by comparing welfare across economies with official contracts that vary in duration and recovery. We find that longer duration official debt tends to deliver higher welfare than alternative short-term liquidity type contracts that are prevalent in the data.

5.1 Specification and Parameterization

The utility function is $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The potential endowment follows a log-normal AR(1) process $\log(z) = \rho \log(z_{t-1}) + \sigma_z \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0,1)$. We discretize this process into 13 different states following Tauchen (1986). The output costs of default are increasing and convex in partial default and the potential endowment, as in Arellano, Mateos-Planas, and Ríos-Rull (2023). These costs are realized only when z is higher than the mean $\bar{z}$, and are linearly increasing in z with slope ϕ. Similarly, output costs are increasing and convex in partial default d_f and d_b, with a slope parameter λ and a curvature parameter γ. The specific functional form for the output cost of default is given by: $y = z(1 - \lambda d_f^\gamma)(1 - \lambda d_b^\gamma)(1 - \mathcal{I}_{d,z}\phi(z - \bar{z}))$, where the indicator function equals one when partial default for either debt is positive, $d_f > 0$ or $d_b > 0$, and $z > \bar{z}$.

We calibrate the model at an annual frequency. We set some parameters to values from the literature and estimate others in a moment-matching exercise. We set the annual international risk-free rate to 2%, consistent with yields from U.S. Treasury bills, and set the coefficient of risk aversion to 2, a standard value in the literature. The autocorrelation of the endowment process is set to 0.87 consistent with our estimates for our panel of emerging markets. We also set the default cost exponent γ to 2, a value close to that estimated in Arellano, Mateos-Planas, and Ríos-Rull (2023), for computational simplicity as it delivers a closed-form expression for partial default.

In computing our model, we also incorporate discrete taste shocks following Dvorkin et al. (2018). These shocks slightly perturb the borrowing decision to achieve numerical stability and robust convergence in the computational algorithm. The parameter ϱ governs the relative importance of the taste shocks for the choice of b' and is set to $5e^{-5}$, which is the smallest value that guarantees convergence in the model.

The rest of the parameters are chosen to best fit moments in the data. The parameters that differ across the debt contracts are $\mu^a, \kappa^a, \vartheta^a$ for $a \in \{b, f\}$. It turns out that in our model given a decay parameter ϑ^a, the parameters that control default on legacy and the accumulation of defaulted coupons, μ^a and κ^a, matter only as the linear combination $\kappa^a - \vartheta^a \mu^a$ in the sovereign program. In practice, we set $\mu^a = 0.18$ for both types of bonds based on estimates by Stefanescu (2016) for the fraction of sovereign bonds in developing countries that contain acceleration clauses relative to the bonds that contain reverse acceleration clauses, but this choice has no bearing on the results.[10] In the moment matching exercise, we only estimate for each type of debt the decay ϑ^a and the recovery factor κ^a.

Table 4: Parameter Values

Parameters Set Externally	
Risk-free interest rate	$R = 1.02$
Risk aversion coefficient	$\sigma = 2$
Endowment persistence	$\rho = 0.87$
Default cost exponent	$\gamma = 2$
Taste shock	$\varrho = 5e^{-5}$
Parameters Set Internally	
Endowment volatility	$\sigma_z = 0.052$
Discount factor	$\beta = 0.954$
Debt contracts	
Decay parameters	$\vartheta^f = 0.907$, $\vartheta^b = 0.794$
Net recovery factors	$\kappa^f = 0.11$, $\kappa^b = 0.19$
Default Costs	
Cost based on partial default	$\lambda = 0.06$
Asymmetric endowment	$\phi = 0.8$

10. Stefanescu (2016) uses the Thomson One dataset covering 882 bonds in developing countries from 1990-2013 and tabulates the use of acceleration and reverse acceleration among these bonds, corresponding to 77% and 59% respectively.

We perform a moment-matching exercise and estimate eight parameters. These parameters are the standard deviation of the potential endowment, the discount factor, the debt contract parameters, and the default cost parameters. We collect these parameters in $\Theta = \{\sigma_z, \beta, \vartheta^f, \vartheta^b, \kappa^f, \kappa^b, \lambda, \phi\}$. We target the volatility of output and 9 moments on the distribution of debts and partial defaults. These moments are the means and standard deviations of debt-to-output ratio for total debt, official debt, and private debt, the mean of partial default, and the mean of the ratios of debt service to output for total, official, and private debt. As in the data, partial default is measured as total debt in arrears due relative to total debt due, $\frac{d^f R^f f + d^b R^b b}{R^f f + R^b b}$. Table 4 shows all the baseline calibration values for the parameters of the model.

All parameters affect all moments, but some moments are more informative of certain parameters. The volatility of output maps into the volatility the shock. The level of total debt informs the default cost parameters, as higher default is associated with higher debt capacity. The net recovery parameters matter for the relative levels of private and official as well as their standard deviations. The discount factor and the default cost parameters also matter for the average partial default and the volatility of the debts. The ratios of debt service to output are informative on the decay parameters.

The resulting parameters controlling official and private contracts, namely $\{\vartheta^f, \vartheta^b, \kappa^f, \kappa^b\}$, imply two properties that are also consistent with external estimates. First, the exercise implies that the durations of official and private debt are 9 and 4.5 years, which are consistent with estimates in Arellano and Ramanarayanan (2012). Second, official debt has lower recoveries; the estimated parameters imply that the recoveries, namely $\kappa^a/(R^a + \mu^a \vartheta^a)$, are on average 41% for official debt and 52% for private debt. The more concessional nature of official debt is consistent with the findings of Schlegl, Trebesch, and Wright (2019). They document an average of 40% recovery across 414 restructurings with official creditors from the Paris Club since 1978 and an average of 60% recovery across 187 restructurings with private creditors, in line with our results.

5.2 Baseline Quantitative Results

We start with the results from our moment matching exercise. Table 5 reports the model's implications for our target moments as well as for additional moments. The model statistics come from a long simulation of 200000 periods, after discarding the first 10000 observations. The model matches well the mean total debt and the breakdown of official and private. In the model debt to output is on average 34%, very similar to the data mean which is 33%. About two-thirds of that debt is official both in the model and data. Debt-to-output ratios are volatile with comparable magnitudes between data and model. The

Table 5: Model Fit (10 moments, 8 parameters)

	Data	Model
Targeted moments		
Total debt		
Mean	33	34
Std. dev.	18	18
Official debt		
Mean	20	21
Std. dev.	12	12
Private debt		
Mean	13	13
Std. dev.	8	6
Debt service to output		
Official	1.6	1.7
Private	1.9	2.3
Partial default	32	28
Output std. dev.	11	12
Other moments		
Debt service to debt		
Official	11	9
Private	19	20
Corr (d^f, d^b)	62	98
Corr (f, b)	42	91

mean partial default is on average close to 30% in the model and data. Finally, in the model, the ratios of debt service to output are equal to 2% for each type of debt. These ratios are very similar in the data.

The table also reports some the correlations between official and private partial default and between official and private debt as additional moments. Both of these correlations are positive, as implied by the model, although in the model these correlations are stronger.

5.3 Policy Rules and Dynamics

Before confronting our model to additional patterns in emerging market data, we describe more details of the workings of the model by illustrating policy rules and dynamics.

Default and debt policies depend on the states, namely (f, b, z). Panel (a) in figure 5 illustrates the policy functions for partial default and debts. The figure is constructed for the mean z; the x-axis denotes the f state and the y-axis is the b state, reported relative to output. The shades are the color map for the partial default policies. The gray area corresponds to no default on either official or private debt, $d^f = d^b = 0$. The white area corresponds to $d^f = d^b = 1$. The blue area has positive partial default; partial default on official (private) debt tends to be more elevated for higher values of official (private) debt.

As is standard, partial default tends to increase with debt. Interestingly, however, default incentives in our model vary with the type of debt. The sovereign can sustain more official debt without defaulting relative to private debt. The sovereign starts to partially default when official debt is above 60% of output for low levels of private debt, while that threshold is about 50% for private debt. The higher debt capacity of official debt is related to the theoretical results in Section 4. Moreover, partial default is higher with a portfolio that is tilted to one type of debt. As illustrated in the figure, the sovereign can sustain about 95% of total debt without defaulting for this shock when it holds a portfolio of about 55% official and 40% private, while it cannot sustain those levels of debt when holding only one type of debt.

We now turn to the debt policies. In our model, as in Arellano, Mateos-Planas, and Ríos-Rull (2023), the dynamics of debts are governed by inter-temporal consumption smoothing incentives, as well as the shape of the bond price functions and default costs. To illustrate these forces, we assume all functions are differentiable and derive the following Euler conditions to the sovereign problem, (where for simplicity assume that $\kappa^a = \mu^a = 0$ for $a \in \{f, b\}$) ,

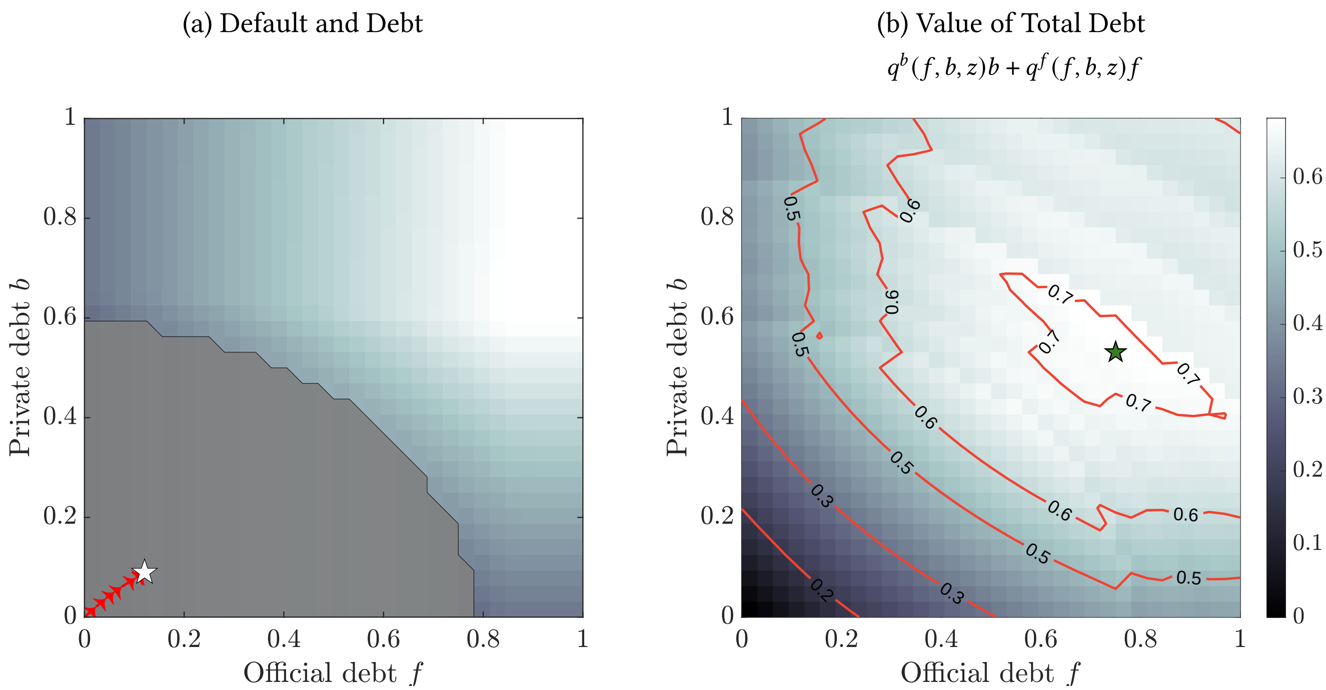
$$u_c\left(q^b + \frac{\partial q^b}{\partial b'}(b' - \vartheta^b b) + \frac{\partial q^f}{\partial b'}(f' - \vartheta^f f)\right) = \beta \mathbb{E}\left[u_c'\left((1 - d^{b\prime})R^b + \vartheta^b q^{b\prime} - \frac{\partial d^{b\prime}}{\partial b'}(z'\psi'_{d^b} + R^b b') - \frac{\partial d^{f\prime}}{\partial b'}(z'\psi'_{d^f} + R^f f')\right)\right]$$

$$u_c\left(q^f + \frac{\partial q^f}{\partial f'}(f' - \vartheta^f f) + \frac{\partial q^b}{\partial f'}(b' - \vartheta^b b)\right) = \beta \mathbb{E}\left[u_c'\left((1 - d^{f\prime})R^f + \vartheta^f q^{f\prime} - \frac{\partial d^{f\prime}}{\partial f'}(z'\psi'_{d^f} + R^f f') - \frac{\partial d^{b\prime}}{\partial f'}(z'\psi'_{d^b} + R^b b')\right)\right]$$

The first condition is the Euler equation for private debt and the second condition is that for official debt, and these govern the dynamics of debt. The left-hand-sides are the marginal benefits of debts, which take into account that bond prices react to bond issuances and the change in prices also affects the value of the legacy debt. Increased private (official) borrowing is beneficial if its price is high and the elasticities of both prices q^b, q^f with respect to private (official) debt are low. High legacy debts increase borrowing incentives because the decrease in prices dilutes the value of this debt. The right-hand-sides are the marginal costs of default and depend on the payoff of future debts ($(1 - d^{a\prime})R^a + \vartheta^a q^{a\prime}$ for $a \in \{f, b\}$) as well as marginal default costs. Private (official) borrowing is more beneficial when the costs from the additional marginal partial defaults are low. Note however, that when partial default is interior it will be chosen in the next period to equalize the marginal costs and benefits of defaults (and satisfy (7)), which makes the terms with the default costs on the right-hand-sides equal to zero.

The red path with arrows in Panel (a) of Figure 5 plots the dynamics of debts when the economy starts with zero debt and z is always at its mean. The white star in the figure is the point that the economy settles at given this shock. Given that the sovereign is impatient relative to the risk-free rate, the sovereign frontloads consumption and borrows. It settles at a point with about 12% of official debt and 9% of private debt, a point of no partial default. This stationary portfolio is tilted towards official debt because of the higher debt capacity of this debt which is encoded in the elasticities of the bond price functions.

Figure 5: Default, Debt Dynamics, and Bond Prices



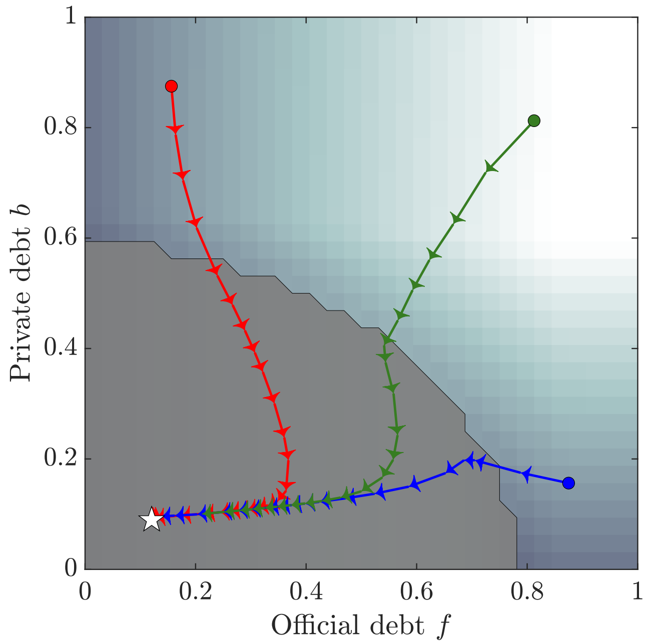
Notes: Panel (a) plots a heat map for partial default for the mean z across the debt states $\{f, b\}$. The gray area corresponds to full repayment; the white area corresponds to full default; the blue areas indicate partial defaults, with lighter colors indicating more intense defaults. The red arrows are the dynamic paths debts starting from zero. The white star is the stationary point for debts when the economy remains at mean z. Panel (b) plots a heat map for the total debt value ($q^b(f,b,z)b + q^f(f,b,z)f$) for mean z across the debt states $\{f, b\}$. Lighter colors indicate higher debt values. The green star is the peak of the debt Laffer curve.

Partial default and borrowing decisions in turn affect bond price functions, which we turn to next. As in many sovereign default models, bond prices tend to fall with larger debts and low endowment shocks because these are the states where the sovereign defaults more. In Panel (b) Figure 5, we summarize these bond price functions with a heat map for the total resources borrowed, namely the total value of debt $q^b(f,b,z)b + q^f(f,b,z)f$, across the set (f, b) given mean z. Lighter colors in the figure correspond to higher values. The plot also illustrates with the red contour lines how the various combinations of (f, b) can give the same resources borrowed to the sovereign. Starting at the origin, increasing debts tend to increase the total resources borrowed but they are capped by a peak. The peak of the debt Laffer curve is illustrated by the green star: this is the maximum amount of resources that a sovereign can get when it starts with zero debt and is achieved with a portfolio that is tilted towards official debt. Note also that portfolios tilted towards official debt need less private debt to reach a total amount of resources borrowed than vice-versa: to get total resources borrowed of 60% of output requires 60% of official and 20% of private, or 60% of private and 30% of official. Of course, the shape of these functions crucially affects the choices of official and private debt, as seen in the first-order conditions presented above.

Exiting Defaults. In our partial default model, bond price functions and borrowing incentives are also at play when exiting default episodes. We now illustrate the forces behind these dynamics. Figure 10 plots the debt dynamics for the economy, when it starts at three different points with high levels of debt. When the sovereign is highly indebted, partial default is high; the sovereign deleverages to exit default and reach its stationary point. As the figure shows, the deleveraging process uses actively a portfolio of official and private debt and it does not feature a monotonic decrease of both debts. The paths in the figure also illustrate that the sovereign tends to issue official debt to reduce private debt at a faster rate. The three different starting points of the debt converge to a path that starts with relatively high official debt and low private (about .4 official and .12 private). Eventually, the sovereign also reduces the official debt to settle at the stationary point. These dynamics show that official debt is useful for the sovereign to exit faster the default episode. We will return to this point with counterfactual analysis below. These deleveraging dynamics are shaped by the bond price functions as well as default costs; the sovereign deleverages to reduce default costs and issues official debt because of more favorable bond price functions.

It is interesting to compare these dynamics to those in Aguiar et al. (2019). A main result of that paper is that the sovereign decreases only the short-term debt to exit the crisis zone and reach the no-default risk zone, without touching the long-term holdings. In that paper, this was because of the worse properties of long-term debt in terms of dilution. In our model, in contrast, the long-term bond which is the official debt, has higher debt capacity, and therefore the sovereign actively uses it in this process.

Figure 6: Exiting Default



Notes: This figure plots a heat map for partial default for the mean output across the debt states $\{b, f\}$ as well as dynamics of debts starting from 3 different points. See the notes to Figure 5.

5.4 Official and Private Debt During Defaults

We now confront our model with additional empirical patterns in emerging markets. A main finding in Section 2.2 is that official debt tends to increase by more during defaults relative to private debt. We explore the model patterns for total, official, and private debt-to-output as well as partial default, and private spreads across bins with and without partial default in the model and data. In Table 6, we present the results; the *No default* bin corresponds to the observations with zero partial default, whereas the bin labeled *Partial default* corresponds to periods with positive values for partial default.

Table 6: Moments Conditional on Partial Default

	Data		Model	
	No default	Partial default	No default	Partial default
Debt to output	24	44	21	44
Official	13	29	13	27
Private	11	15	8	17
Private spreads	4	8	1	5
Partial default	0	32	0	28

The model generates higher debt-to-output ratios with partial default as in the data. During periods of positive default, total debt-to-output is 44% while with no default it is 21%. These patterns are similar to those observed in the data, with levels of 44% vs. 24% in periods of positive and no partial default. Official debt increases more with default than private debt in both model and data. In the model, however, these differences across debts are a bit less accentuated than in the data. Spreads also increase with default. Spreads on private debt are about 4% higher when partial default is positive in the model and the data.

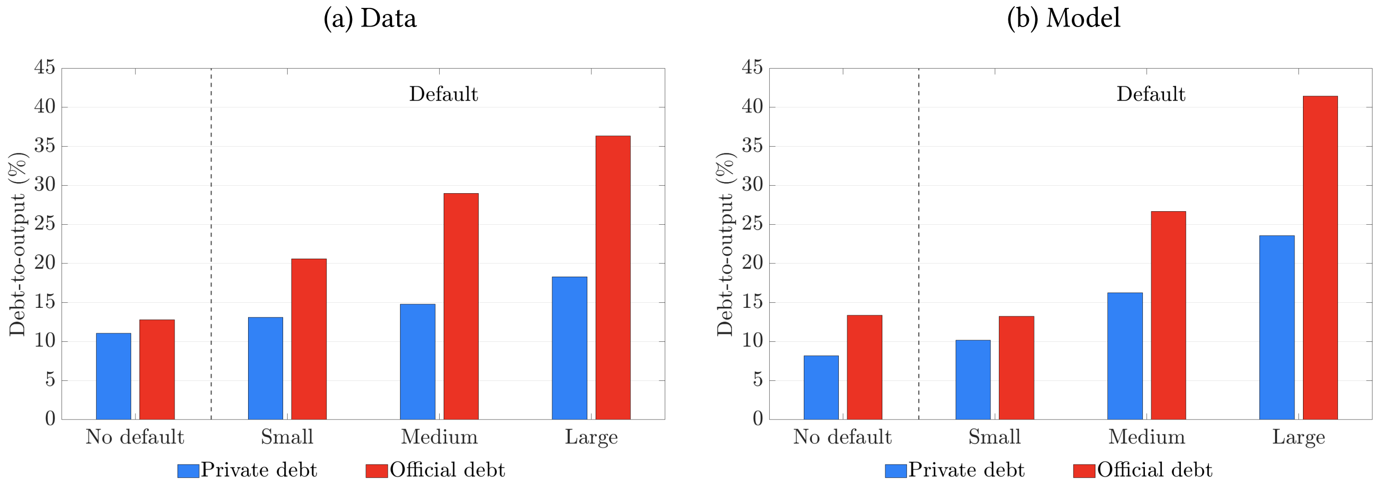
As in the empirical analysis, we further examine the patterns of official and private debts across finer partial default bins. In Figure 7, we report mean official and private debt across 4 bins, the same ones we used in Section 2.2. As before, the bars represent the averages of official and private debt-to-mean output across bins; the red bars correspond to official debt and the blue bars correspond to private debt; the left panel is the data and the right panel is the model. The figure illustrates that in the model, as in the data, debt increases monotonically with partial default. When partial defaults are small, under the 25 percentile of the distribution, debt to output is higher than when default is zero. Large defaults, over the 75 percentile of the distribution, feature the highest debt-to-output. The increase is more accentuated for official debt than for private debt in both the model and data.

This analysis illustrates that the model captures well the conditional patterns of debts across partial default. Periods of more intense defaults are associated with higher debt-to-output, especially so for official debt. In the model, periods of defaults are associated with a history of low endowment realizations and build-up of debt. The sovereign shifts its portfolio towards official debt as this debt offers greater debt capacity. We expand on these dynamics next, by analyzing patterns dynamics within default episodes.

Default Episodes. We now study default episodes. As in the data, we use the simulated time series data to track default episodes: a period of time with continuous positive partial default. Default in the model is persistent, which gives rise to long default episodes. The average length of default episodes in the model is 10 years which is the same length found across episodes in the data.

We study how debts, partial defaults, and spreads evolve within default episodes. Table 7 reports the results of the model and compares them to data. During default episodes, total debt features a hump-shaped pattern in the model, as in the data. In the model, total debt to output increases about 8% from the year before the episode up to the middle of the episode (i.e. 37% minus 29%), which is similar to the 7% increase observed in the data. In the model, official debt increases more than private debt, as in the

Figure 7: Official and Private Debt across Default Bins



Notes: The figure plots mean debt (relative to mean output) across default quartiles in the limiting distribution for the models with official and private debt.

data. The contribution of private debt to this total increase is, however, somewhat more pronounced in the model relative to the data.

The model dynamics around episodes occur in response to a sequence of shocks and endogenous debt dynamics, where the sovereign increases and then decreases its debts. The sovereign enters the default episode when it receives an adverse endowment shock. As the shock recovers, the sovereign deleverages and exits. As explained above, the sovereign uses more heavily official debt during the episode because of a more lenient bond price function, which reflects the higher official debt capacity. As the sovereign exits the episode, it reduces private debt more aggressively than official debt. This effect is seen by comparing the levels after the episode; here the private debt level is very comparable to that before the episode, while the official debt level continues to be elevated.

Interestingly, the dynamics in the data are consistent with these forces. Official debt increases more in the ramp-up of debt of the episode, and private debt is reduced more rapidly, relative to official debt. In fact, in the data, this effect is accentuated as the level of private debt after the episode is below that seen before the episode, while official debt after the episode continues to be elevated. Through the lens of our model, the more aggressive deleverage of private debt results from its lower debt capacity and higher propensity to lengthier defaults.

Table 7: Default Episodes

	Dynamics of Debt			
	Before	Beginning	Middle	After
Data				
Total	33	35	40	33
Official	17	18	23	19
Private	16	17	17	14
Model				
Total	29	32	37	33
Official	18	20	23	21
Private	11	12	14	12

Notes: The mean length across default episodes is 10 years in the data and in the model. See the notes in Table 3 for further details.

6 Counterfactuals

We now use our baseline quantitative model to perform counterfactuals. In the first counterfactual, we evaluate the feasibility of voluntary swaps across official and private debt, that generate Pareto improvements for lenders and the sovereign. We find that swaps are feasible for a large region of the state space and can generate sizable welfare gains. Swaps tend to reduce default and spreads and tend to reduce private debt. Next, we study the design of official debt contracts by comparing our baseline with economies that feature official debt that is shorter-term and less concessional. This experiment is motivated by various liquidity programs from multilateral organizations. We find that short-term official debt offers limited welfare benefits and that the best design instead consists of long-duration bonds with few concessions.

6.1 Voluntary Swaps

In this section, we evaluate the feasibility of voluntary swaps, where the country and its official and private lenders agree to a debt exchange. A voluntary swap is feasible if such an exchange increases the value of the sovereign and the value of the debt to all lenders. Given a state (f, b), a feasible swap exists if there is a pair $(\hat{f}, \hat{b})$ such that the following two conditions hold:

$$V(\hat{f}, \hat{b}, z) \geq V(f, b, z) \tag{9}$$

$$H(\hat{f}, \hat{b}, z) \geq H(f, b, z), \tag{10}$$

with at least one with strict inequality and where $H(f, b, z) = H^f(f, b, z)f + H^b(f, b, z)b$, the sum of the values of debt to official and private lenders, defined in Section 3.2. Conditions (9) and (10) indicate that the value of the sovereign and of lenders must increase with the swap program. If a voluntary swap exists, then it constitutes a Pareto improvement because, under the new contract, the sovereign and the lenders are better off.

To understand why states with feasible swaps may arise in equilibrium it is useful to consider small changes in debts. Swaps are feasible on the margin in state (f, b) if the total differentials are positive $V_f df + V_b db > 0$ and $H_f df + H_b db > 0$ for some small changes db, df. We can relate the marginal values of the sovereign and lenders using the sovereign optimality conditions and the bond price functions. By combining the lenders' values in (6) with the first order conditions with respect to b' and f', and considering a small deviation around the optimal choices, we can get that at the optimal portfolio the following condition is satisfied

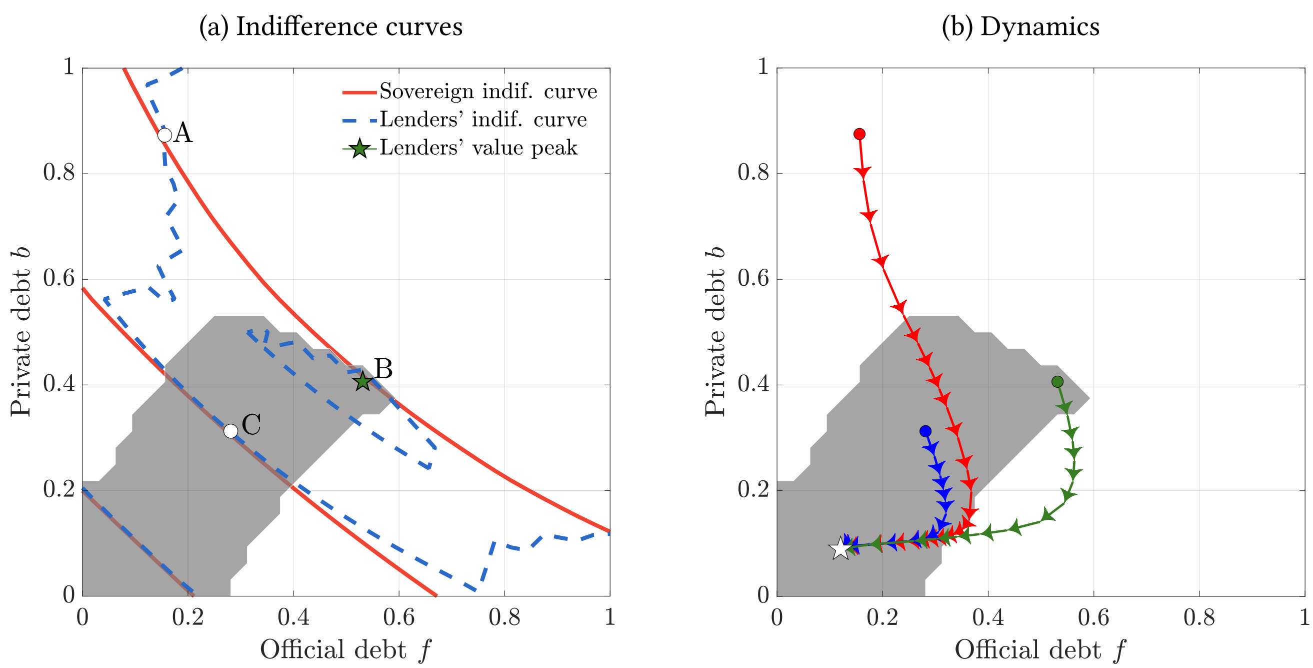
$$\mathbb{E}\big[R\ \underbrace{(H_{b'}db' + H_{f'}df')}_{\text{gain lenders}}\big] + \frac{\beta}{u_c}\ \mathbb{E}\big[\underbrace{V_{b'}db' + V_{f'}df'}_{\text{gain sovereign}}\big] = \underbrace{\vartheta^b b\left(\frac{\partial q^b}{\partial b'}db' + \frac{\partial q^b}{\partial f'}df'\right) + \vartheta^f f\left(\frac{\partial q^f}{\partial b'}db' + \frac{\partial q^f}{\partial f'}df'\right)}_{\text{gain value of legacy debt}} \tag{11}$$

This portfolio condition says that at an interior optimal the expected marginal gain for lenders is negatively related to the expected marginal gain for the sovereign and positively related to the gain in the value of the legacy debts, with a small deviation $\{db', df'\}$. Consider now an example with no uncertainty and a particular deviation $\{db', df'\}$ that gives the sovereign positive welfare gains, $V_{b'}db' + V_{f'}df' > 0$. Note that given that the value of the sovereign decreases with each type of debt, this deviation requires a reduction in at least one type of debt, $db' < 0$ and/or $df' < 0$. If the sovereign has a state of no legacy debt, $b = f = 0$, then the portfolio condition under no uncertainty immediately says that the sovereign would never choose a portfolio with feasible swaps ex-post. Any deviation ex-post that increases the value to the sovereign necessarily decreases the value to lenders: the RHS in (11) is zero and $u_c > 0$. This implies that legacy debt plays a key role in the feasibility of swaps along the equilibrium. With legacy debts, deviations that increase sufficiently their value could give gains to both the sovereign and the lenders. In addition, with uncertainty, particular ex-post realizations could also give rise to feasible swaps ex-post for insurance reasons.

This analysis relates to Hatchondo, Martinez, and Padilla (2014), that illustrate that the sovereign may borrow beyond levels that maximize the value of debt to all lenders as the value of the new loans may increase with a reduction in the value of the legacy debt. In our model with two types of debts, private or official loans may dilute the value of both private and official legacy debts and debt accumulated as arrears. In our model, Pareto improvements may arise not only because the economy has too much debt, but also because they have an inefficient portfolio of debts.

We now explore the feasibility of swaps in our baseline model. We find that across the state space of the baseline model, there are many states (f, b, z) where swaps are feasible. In Figure 8, we illustrate the set of (f, b) where swaps are feasible given middle z; in the white area of panels (a) or (b) swaps are feasible, while in the grey area they are not. The state space features feasible swaps when the sovereign has too much debt, the north-east region of the state, or when it has a portfolio that is tilted too much to one type of debt, the north-west and south-east regions of the state.

Figure 8: Swaps



Notes: The white area in the figures are the states $\{f, b\}$ given mean z with feasible swaps, where conditions (9) and (10) are satisfied. In the grey area, swaps are not feasible. Panel (a) plots with solid lines indifference curves for the sovereign and in the dash lines indifference curves for lenders. The green star is the peak of the lenders' value. A swap from point A that maximizes lenders' value is point B, and one that maximizes the sovereign welfare is point C. Panel (b) plots the debt dynamics starting from points A, B, and C.

Panel (a) of Figure 8 also plots the indifference curves for lenders and the sovereign across the portfolio of debts.[11] The lenders' indifference curves increase towards the star of the figure, which is the peak for lenders, while those for the sovereign increase towards the origin. Importantly, the lenders' indifference curves are more convex in the portfolio than the indifference curves of the sovereign and these relative convexities open the door for voluntary swaps. Consider for example the state corresponding to point A in the Figure. In this point, there is a set of feasible swaps that result in various combinations of welfare gains for the lenders and sovereign. For example, a swap from point A to point B, which corresponds to the peak, maximizes the gains for lenders, while keeping the sovereign at the same welfare level. Here lenders gain about 44% of their initial value (or 22% of mean output). A swap to point C, in contrast, maximizes the welfare of the sovereign while keeping the value to lenders unchanged. This swap results in a consumption equivalence gain of 1.3%.

We find that in general swaps that maximize the lenders' value tend to be higher in regions of the state with portfolios tilted to one type of debt, with lenders gaining about 25% of output. Swaps that maximize the country tend to be higher with portfolios that contain high amounts of both debts. The highest gain in the state space is when the sovereign holds 200% of debt equally split between official and private, with gains of about 3.5% of consumption equivalence.[12] In the limiting distribution, however, the economy visits states with feasible swaps about 5.3% of the time and welfare gains are modest. Across these states, the welfare gains from swaps that maximize the lenders' value are on average 1.1% while the gains if swaps maximize the sovereign are on average 0.04% of consumption equivalence.[13]

To understand how a swap affects the economic outcomes, in panel (b) of the figure, we illustrate the debt dynamics after a swap. The red path is the original deleveraging path starting from point A, which we analyzed above in Figure (10). The green and blue paths are the ones that result after implementing swaps that maximize lenders and the sovereign respectively, that is swaps from A to point B and C. After the swaps, private debt decreases and settles at a lower level over time. Official debt, in contrast, tends to first increase and then decrease. As before, the increase is official debt helps the sovereign reduce faster private debt. Note that with these swaps, the new states on impact are in the grey region, but in the subsequent dynamics the economy travels through states where swaps are feasible. As explained above through the portfolio condition (11), the sovereign may choose in equilibrium debt levels with

11. The indifference curves of lenders are similar to those presented above in panel (b) Figure 5, only that here they are ex-post values.

12. In Figure 10 of the appendix, we present heat maps of the gains across the state space for mean z that illustrate these findings.

13. These modest welfare gains are expected as our baseline model only considers business cycle shocks and abstracts from more extreme shocks like a pandemic or natural and economic disasters.

feasible swaps due to the dilution incentives for legacy debts; these dynamics illustrate these forces at play. Also, importantly, point A in the baseline model is associated with defaults in private debt and high private spreads. Swaps to points B or C eliminate the default and reduce spreads, which constitute a source of welfare gains.

These results illustrate the feasibility of Pareto improvements that swap one type of debt for another. The literature has argued that the implementation of debt swaps can be notoriously difficult. Bulow, Rogoff, and Dornbusch (1988), for example, argues that countries may not increase welfare with debt buybacks, because the gains that legacy debtors get may rip any gains to the country. The official lenders in our setup could be very useful in the implementation of these swaps because they can help coordinate it. In our model, when the sovereign has a lot of private and little official debt, the official lender could buy the private debt in secondary markets and reach an agreement with the sovereign of an increase in official debt due. In contrast, in states with a lot of official and little private debt, official lenders could reach an agreement with the country that reduces the official debt of the country in exchange for obtaining the proceeds of a new auction that the country issues with private lenders.

6.2 Design of Official Debt

In the baseline model official debt differs from private debt in that it is of longer duration and carries a lower recovery factor, deeming it more concessional. We now study how these properties affect the economy and the implications for welfare. To this end, we perform comparative statistics to the parameters ϑ^f and κ^f which change the duration and recovery of official debt. We show that official debt of longer duration tends to be beneficial to the sovereign. Higher recovery factors tend to benefit the lenders and benefit the sovereign mostly in low-debt states. We also find that these welfare differences are linked to how different economies bear the costs associated with deleveraging episodes as they exit defaults.

In Table 8, we report the main summary statistics across various comparative statistics and compare them to the baseline model. Column (2) lowers official debt duration from 9 to 7 years. Column (3) makes official debt more concessional by reducing the recovery factor of official debt to 30%. Column (4) makes official debt have identical duration and recovery to private debt, of 4.5 years and 54% respectively. In column (5), official debt has a duration of 2 years and a higher recovery factor of 80%. We label this setting a “Multilateral” because it resembles certain official loans from multilateral organizations that are quite short-term and with very few concessional characteristics such as the Stand-By Agreement

(SBA) or the Short-Term Liquidity Line (SLL) of the IMF and the Federal Reserve swap lines. In column (6), we analyze the case of longer and less concessional official debt, of 13 years and 70% recovery.

We describe first the top panel of Table 8. We find that official debt with shorter duration or lower recovery features lower debt capacity, as seen by the comparative statics in columns (2) and (3), respectively. In these economies, the average official debt to output is reduced by about 5%. These characteristics also lower default costs but increase consumption volatility. Column (4) features official debt with equal characteristics as private debt, namely shorter duration and lower recovery. Here we see that these forces are compounded, with further reductions in official debt capacity, reduced default costs, and higher consumption volatility. Column 5, our multilateral economy, displays outcomes of two offsetting forces; shorter duration reduces debt while higher recovery increases debt, which in net leads to a sizable level of official debt. Consumption volatility, however, is fairly elevated in the economy, but default costs are reduced. Finally, in column (6), the economy's official debt has the longest duration and high recoveries, and it features the highest debts and default costs, but a sizable decline in consumption volatility. Across these experiments, we also find that settings that increase official debt substantially also tend to increase modestly private debt.

The lower panel of the table contains welfare comparisons across these economies. Welfare for the sovereign is the consumption equivalence in each counterfactual economies relative to that of the baseline given a state, namely $((V^{\text{counter}}(f,b,z)/V(f,b,z))^{1/(1-\sigma)} - 1) \times 100$. Welfare for lenders is the difference in values given a state in percent of the mean endowment, namely $(H^{\text{counter}}(f,b,z) - H(f,b,z))/\bar{z} \times 100$. We report results for the mean endowment in states with zero debt, the mean debts of the baseline economy, and high levels of debt in the baseline, which here are 50% and 25% for official and private debt respectively.

We find that welfare for the sovereign tends to be lower in economies with shorter-term official debt. As seen in column (2), welfare is especially lower in states of high debt. In contrast, the welfare implications of varying recoveries for the sovereign is highly dependent on the state. As seen in column (3), the sovereign benefits from lower recoveries in high-debt states, but these are costly for low-debt states. Lower recoveries help in high-debt states because default here reduces the burden of debt, and it is discharged at higher rates, while for low debts the lower debt capacity of contracts with lower recoveries is detrimental. In the settings in column (4), the shorter duration effects dominate, and therefore welfare is qualitatively similar to column (2). In the multilateral economy of column (5), in contrast, the recovery effect dominates: higher recovery is beneficial with low debts, and costly in high

Table 8: Counterfactual

	Baseline	Official Debt				
		Shorter	Lower recov.	Shorter + Lower recov. (Equal to Private)	Shorter + Higher recov. (Multilateral)	Longer + Higher recov.
	(1)	(2)	(3)	(4)	(5)	(6)
Official debt	21	16	15	12	16	97
Private debt	13	13	13	12	11	16
Partial default	28	24	24	21	21	55
Consumption std. dev.	0.92	0.93	0.93	0.95	0.95	0.81
Default costs	0.62	0.51	0.50	0.39	0.35	1.9
Welfare (%)						
Sovereign: Consumption Equiv. Gain						
No debts	0.00	-0.002	-0.02	-0.0003	0.13	0.04
Mean debts	0.00	-0.03	0.004	-0.06	0.006	0.13
High debts	0.00	-0.07	0.06	-0.19	-0.41	0.17
Lenders: Change in Value						
No debts	0.00	0.00	0.00	0.00	0.00	0.00
Mean debts	0.00	0.65	-0.84	1.66	4.62	0.52
High Debts	0.00	0.78	-2.71	1.38	12.38	5.28

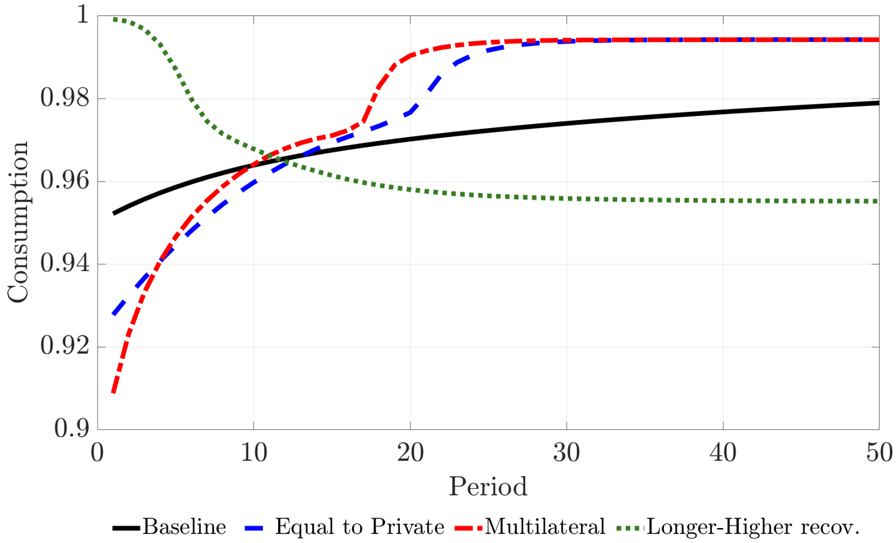
Notes: This table reports the main statistics and welfare comparison across comparative statistics across the duration and recovery of official debt, controlled by the parameters $\{\vartheta^f, \kappa^f\}$. Column (1) contains the results for the baseline calibration, with official duration of 9 years and recovery of 40%. Columns (2) and (3) report results comparative statics on ϑ^f and κ^f, which reduce official duration to 7 years and recovery to 30%, respectively. In column (4) official duration is 4.5 years and recovery is 52%, making official and private debt equal. In column (5) official has the properties of liquidity facilities from multilateral organizations, duration is 2 years and recovery is 80%. In column (6) official duration is 13 years and recovery is 70%. Welfare for the sovereign is consumption equivalence measures relative to baseline in percent. Welfare for the lenders is reported as the difference in values ΔH relative to the baseline, reported in units of mean output in percent.

debt states due to less debt discharge. Finally, the settings in column (6), with longer duration and higher recovery, have the highest welfare gains, especially for higher debts.

What are the sources of these welfare gains and losses for the sovereign? We find that in our economy the main sources of the gains for the sovereign arise from differences in consumption volatility and also the ability to exit defaults and debt crises without excessively costly deleveraging. On consumption volatility, as shown in the table, economies with lower consumption volatility tend to feature higher welfare. For example, the much lower consumption volatility in column (6) is associated with the highest welfare gains. Moreover, we find that this volatility is correlated with the sensitivity of consumption to endowment shocks. For example, the elasticity of consumption to endowment shocks is 0.97 in the baseline while it is 0.95 in the economy of column (6).

Second, and importantly, the welfare rankings also reflect the consumption costs of deleveraging to exit debt crises, illustrated in Figure 10. Economies with longer loans tend to be able to exit these

Figure 9: Exiting Defaults: Consumption Paths during Deleveraging



Notes: This figure plots the consumption paths for the baseline model (column (1) in Table 8), and counterfactuals (4), (5) and (6) in Table 8 when the economy starts in the mean output and "High debts" state in period $t = 0$. Consumption is reported relative to mean output.

states with smoother consumption profiles. In Figure 9, we plot the consumption paths for the various economies starting in the state of high debts and assuming that the endowment remains at mean z. In the baseline economy, consumption is about 5% lower than the endowment and increases about 3%, as the economy reduces its debt to the stationary levels which are associated with low default risk. In the economy with shorter debt and lower recovery, the consumption decline is about 7% and features a more steep path. The economy with longer duration and higher recovery in contrast can roll over the debt completely and in fact debt increases further over time, given the benefits from tilting consumption. These consumption paths explain why the welfare losses in an economy with shorter-term debt can be sizable in high debt states.

The sources of welfare gains in our model are connected with the findings in Aguiar, Amador, and Fourakis (2020). They show the sources of the welfare differences in the sovereign default models arise due to differences in consumption variability, the ability to tilt the consumption paths, and default costs. Our analysis contains these forces but expands the interpretation of the ability to tilt consumption paths to environments where the sovereign needs to deleverage to exit defaults. We also find as they do, that the benefits from consumption smoothing and tilting dominate the overall effect, while the costs of default play only a modest role and in our setting tend to offset the other sources of welfare differences.

The properties of official debt also affect the welfare of lenders. Although lenders break even in expectation with new issuances, as seen from condition (6), the ex-post value depends on the level of debts, default decisions, recovery values, and continuation prices of the debts which encode future losses from defaults. The duration of official debt has two offsetting effects on the value of lenders. One effect is that given a state, shorter-duration debt is associated with more default, which tends to decrease the values of lenders. Continuation prices, however, tend to be higher with shorter duration because the steep price schedules give the sovereign higher incentives to deleverage. This latter effect is the dominant one in columns (2). The recovery of official debt also directly affects the value of lenders; lower recoveries are associated with lower values for lenders as these loans are more concessional, as seen in column (3). The effects on lenders values in the rest of the comparative statics are shaped by how these forces counteract each other. In column (4), the change in duration is more significant, and therefore it dominates. We do find, however, that for even higher levels of debt, this economy can lead to a loss for lenders because of large debt discharge with default. In columns (5) and (6), the high recovery effect dominates as lenders' benefits with shorter or longer duration debt.

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A Robustness on Empirical Analysis

This appendix provides robustness analysis for the patterns of total, official, and private debt during defaults. Table 9 reports debts related to trend output, constructed from nominal Gross Domestic Product in U.S. Dollars. This table shows that the patterns in Tables 2 and 3 remain unchanged with this different definition of debt. Official debt increases by more during partial defaults relative to private debt and also accounts for much of the hump-shaped patterns of debts during default episodes.

Table 9: Partial Default and Debt to Trend Output

Default Flag and Debts				
	Overall mean	No default	Partial default > 0	
Debt to output (in %)				
Total	32	23	43	
Official	19	13	28	
Private	13	11	15	
Dynamics during Default Episodes				
	Before	Beginning	Middle	After
Partial default	0	14	26	0
Debt to output (in %)				
Total	33	34	39	33
Official	17	18	23	19
Private	16	16	16	14

Notes: See notes to Tables 2 and 3. Debt ratios here are reported relative to trend output. Trend output is constructed with Hodrick-Prescott filter with smoothing parameter of 6.25.

Our baseline analysis decomposes debt between that with official and private creditors, and uncovers different patterns during defaults. Here we analyze patterns within official debt, namely with multilateral creditors and bilateral creditors. Multilateral creditors consist on multilateral organizations such as regional development banks, the World Bank, and the IMF. Bilateral creditors are governments from other countries and include the Paris Club lenders. Table 10 shows that bilateral is about 60% of official debt and that both multilateral and bilateral debts increase with partial default. The patterns across partial default bins and during episodes are similar across these two types of official debt, but the increases are more accentuated with bilateral debt.

Table 10: Partial Default and Official, Multilateral, and Bilateral Debts

Default Flag and Debts				
	Overall mean	No default	Partial default > 0	
Debt to output				
Official	20	13	29	
Multilateral	8	6	10	
Bilateral	12	7	19	
Dynamics during Default Episodes				
	Before	Beginning	Middle	After
Debt to output				
Official	17	18	24	19
Multilateral	7	7	9	8
Bilateral	10	11	15	11

Notes: All numbers are reported in percentage points. See notes to Tables 2 and 3. As described in Table 1, official debt is the sum of PPG debt from bilateral and multilateral.

B Theoretical Derivations

This appendix contains the proofs for Section 4.

Proof of Lemma 1

Consider an economy that satisfies Assumption 1. We start analyzing the case with only private debt. Default policies are:

$$d^b = \begin{cases} 0, & \text{if} \quad (1+r)b \leq z - z_L \\ 1, & \text{otherwise.} \end{cases} \tag{12}$$

The bond price function for private debt depends on the one-period ahead default policy, such that

$$q^b(b) = \begin{cases} 1, & \text{if} \quad (1+r)b \leq z - z_L \\ 0, & \text{otherwise.} \end{cases}$$

The private loan that maximizes the budget is $b_{\max} = \frac{z-z_L}{1+r}$ and the associated price is $q^b = 1$.

Next we consider using only official debt. Default policies in this case are

$$d^f = \begin{cases} 0, & \text{if} \quad rf \leq z - z_L \\ 1, & \text{otherwise.} \end{cases} \tag{13}$$

For this debt, the bond price function depends on all future default incentives and future borrowings, and satisfies $q^f(f) = \frac{1}{R}[(1 - \mathbf{d}^\mathbf{f}(f))r + q^f(\mathbf{f}'(f))]$.

Consider a candidate official debt level $\hat{f}$ that gives the sovereign barely enough incentives to repay in the future $\hat{f} = \frac{z-z_L}{r}$. If the sovereign remains with this level of official debt forever, namely $\mathbf{f}'(\hat{f}) = \hat{f}$, then the default policy and the bond price function imply that $q^f(\hat{f}) = 1$. At time zero, such contract is the maximum transfer that the sovereign can obtain, because it is the one that keeps the sovereign at its indifferent point of defaulting in every period in the future. To complete the proof we show next that in fact it is optimal for the sovereign to remain at $\hat{f}$.

When the sovereign starts at $\hat{f}$, any additional borrowing $f' > \hat{f}$ will necessarily generate default, at least temporarily as the default policy indicates that the following period the sovereign will default. This implies that $q(f) < 1$ for $f > \hat{f}$. Moreover, this price will be non-zero only if the sovereign chooses in the future to deleverage and lower its debt to the non-default region. However, it is never optimal for the sovereign to do so because this implies consumption falling below z_L during the deleveraging phase, with all the gains accruing to the legacy lenders. Given that it is never optimal for the sovereign to deleverage to levels below $\hat{f}$ when its debt is $f > \hat{f}$, then the price is zero in this range, such that

$$q^f(f) = \begin{cases} 1, & \text{if} \quad f \leq \frac{z-z_L}{r} \\ 0, & \text{otherwise.} \end{cases}$$

Given these prices, it is never optimal for the sovereign to borrow beyond $\hat{f}$, as it gets zero from such loans without increases in consumption in future. Moreover, given preferences, it is never optimal to lower its debt when it starts at $\hat{f}$.

Proof of Lemma 2

The debt limits that prevent default equate the values of repayment and default. Given preferences and the fact that the bond price function jumps to zero beyond its limit, it is optimal for the sovereign to always borrowing to the limit. Moreover, the stationary and recursive structure imply that borrowing limits do not depend on time. Therefore, the private debt limit satisfies

$$\sum_{t=0}^{\infty} \beta^t [z - (1+r)b_{\max} + q^b b_{\max}] = \sum_{t=0}^{\infty} \beta^t z_L.$$

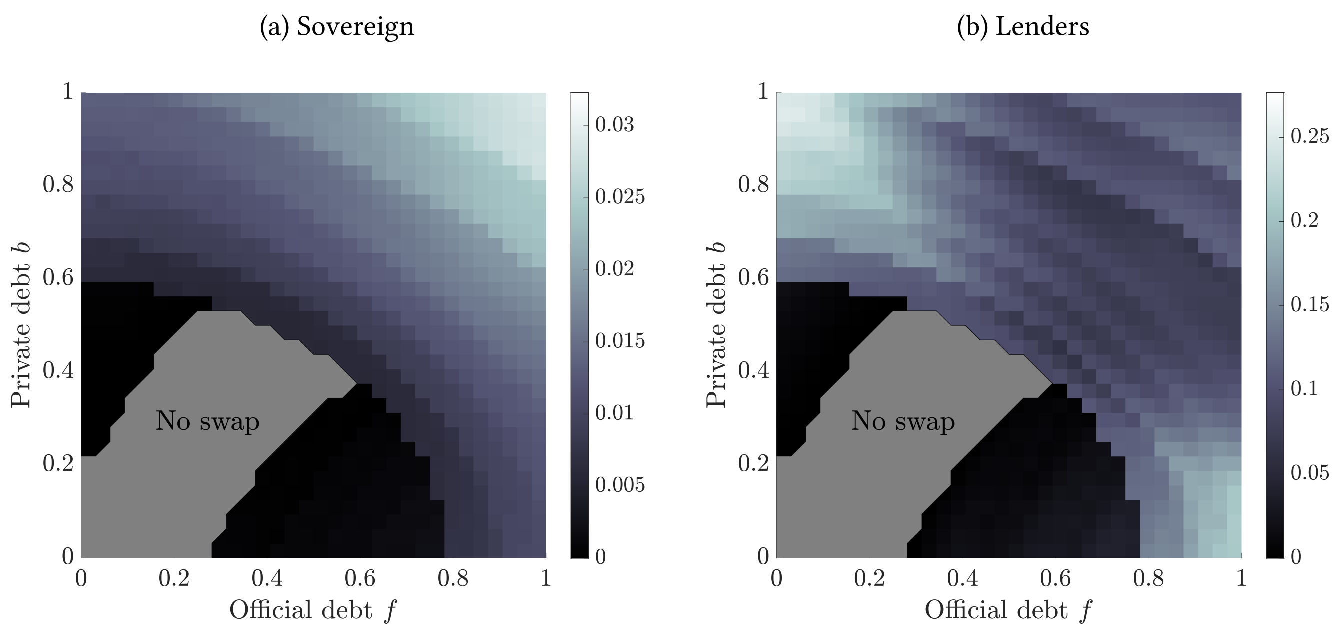
The official debt limits satisfies

$$\sum_{t=0}^{\infty} \beta^t [z - r f_{\max} + q^f (f_{\max} - f_{\max})] = \sum_{t=0}^{\infty} \beta^t z_L.$$

Prices without uncertainty are simple: $q^f = 1$ if $f \leq f_{\max}$ and $q^b = 1$ if $b \leq b_{\max}$. We solve for $f_{\max}$ and $b_{\max}$ using these prices and get the result in the Lemma.

C Swap Welfare Gains

Figure 10: Swap Welfare Gains



Notes: The gray area in the figures are the states $\{f, b\}$ given mean z with no feasible swaps. Panel (a) plots the welfare gains for the sovereign of implementing a swap that maximizes its value, measured in consumption equivalence units. Panel (b) plots the welfare gains for the lenders of implementing a swap that maximizes the total value of debt, reported relative to mean output.