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Leonardo Barreto*†

Abstract

This paper develops a regime-switching newkeynesian model for a small open economy, with an occasionally binding financial friction that allows for endogenous financial crises. The model has two regimes: a regime for normal economic times, in which financial market access is unconstrained, and a crisis regime, characterized by curtailed access to foreign borrowing. The transition probability between regimes depends on the endogenous variables of the model. We employ this framework to analyze the macroeconomic implications of adapting the Inflation Targeting (IT) strategy in a way that takes into account the possibility to prevent the occurrence of financial crises. We calibrate the model using colombian historical data. The results show that monetary policy has major limitations when it seeks to prevent financial crises. As the central bank gives more importance to the GDP growth gap, the frequency with which crises occur decreases. However, this reduction is quantitatively small. On the other hand, as the monetary authority responds more strongly to increases in the external debt growth rate, the frequency with which the economy goes into crisis is not significantly different from the current IT scheme. However, the volatilities of inflation and consumption are much higher.

Keywords: Financial crisis, small open economy, regime-switching, inflation targeting, Colombia.

JEL Classifications: E44, E50, E52, E58.

*Professional, Macroeconomic Modeling Department, Banco de la República, Colombia. cl.barreto15@gmail.com

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Política monetaria no convencional en un modelo con cambio de régimen y crisis financieras endógenas

Leonardo Barreto*†

Resumen

Este documento desarrolla un modelo neokeynesiano de cambio de régimen para una economía pequeña y abierta. Se incorpora una fricción financiera ocasionalmente vinculante que permite la ocurrencia de crisis financieras endógenas. El modelo tiene dos regímenes: un régimen para tiempos normales, en donde hay pleno acceso a los mercados financieros internacionales, y un régimen de crisis, caracterizado por acceso restringido al endeudamiento externo. La probabilidad de transición entre los dos regímenes depende de las variables endógenas del modelo. Se emplea este marco para analizar las implicaciones macroeconómicas de adaptar la estrategia de Inflación Objetivo (IO) de tal forma que tenga en cuenta la posibilidad de prevenir la ocurrencia de crisis financieras. El modelo es calibrado utilizando datos históricos de Colombia. Los resultados muestran que la política monetaria tiene limitaciones importantes cuando busca prevenir crisis financieras. A medida que el banco central le otorga más importancia a la brecha de crecimiento del PIB, la frecuencia con la cual ocurren las crisis disminuye. Sin embargo, la reducción es cuantitativamente pequeña. Por otro lado, a medida que la autoridad monetaria responde con mayor fuerza a incrementos en la tasa de crecimiento de la deuda externa, la frecuencia con la cual la economía enfrenta crisis no es significativamente diferente a la que implica el esquema de IO actual. No obstante, las volatilidades de la inflación y el consumo son mucho mayores.

Palabras clave: Crisis financiera, economía pequeña y abierta, cambio de régimen, inflación objetivo, Colombia.

Clasificación JEL: E44, E50, E52, E58.

*Profesional, Departamento de Modelos Macroeconómicos, Banco de la República, Colombia. cl.barreto15@gmail.com

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1 Introduction

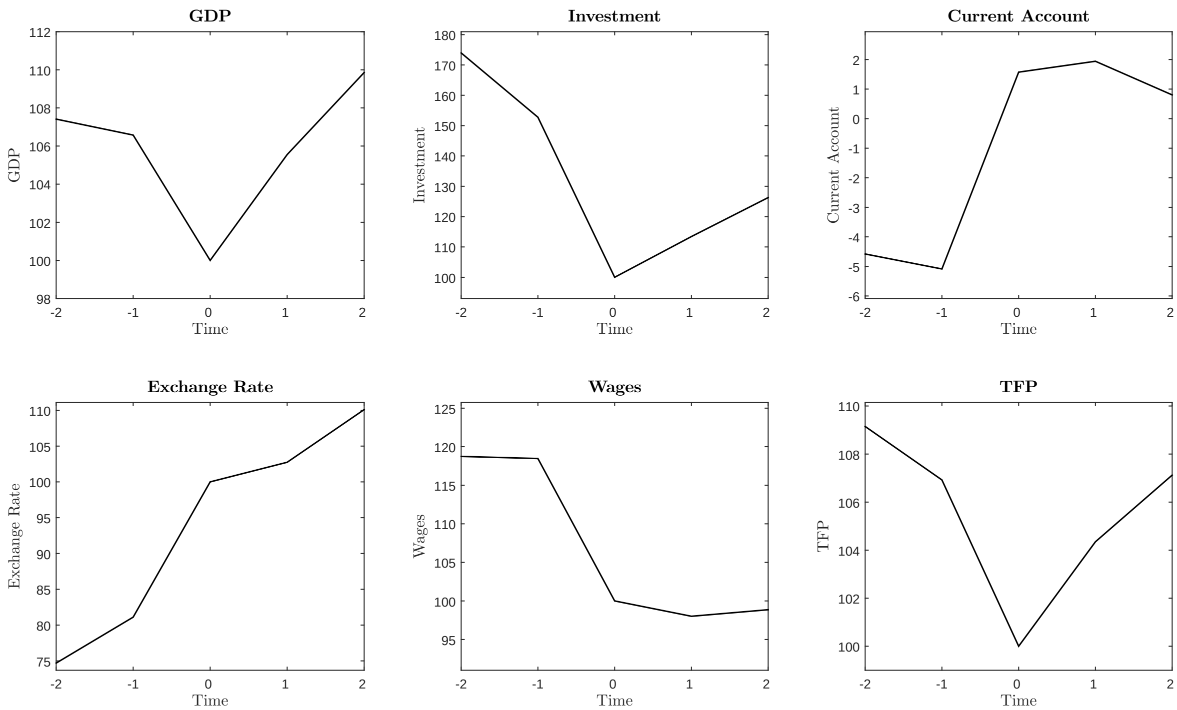
The recent global financial crisis illustrated the importance of financial markets imperfections for emerging and advanced economies alike. Market economies can experience deep recessions that differ markedly from typical business cycles downturns. In particular, the loss of access to world capital markets has played a key role in emerging markets crises, implying sudden adjustments in expenditure plans. Given that these economies are usually net debtors in the international financial markets, higher indebtedness becomes a risk factor in the presence of borrowing constraints to foreign financing. When external debt reaches levels close enough to the limit, typical realizations of exogenous shocks may trigger a financial crisis. Some striking macroeconomic regularities characterize these events: (1) reversals of international capital flows, reflected in sudden increases in the current account, (2) declines in total factor productivity, domestic product and absortion, and (3) increases in aggregate bond spreads. Figure 1 shows some of these facts using five-year event windows, centered on financial crisis ocurring at date $t = 0$. The dating and location follows Calvo et al. (2006) classification.

These events have caught the attention of policy makers and academics as their macroeconomic consequences are always important. Most central bank boards and monetary policy committes have shown willingness to prevent financial crises, and most discussion has tended to focus on the relationship between monetary policy and financial stability. Although the argument that monetary policy decisions should take into account the risk of financial crises had already been put forth before the crisis (e.g. Lowe and Borio (2002)) there is still no consensus among central bankers on the desired strategy for monetary policy.

The literature has proposed several arguments in favor and against modifying inflation targeting (IT) doctrine, which actually has been adopted by most central banks as a platform for guiding their monetary policy decisions[1]. Among the proponents of the thesis that IT should not be modified, they have argued that even though it may be desirable to head off financial crises, such events are simply not predictable enough, and in that sense, central banks should not try to "lean against" financial risks. As Gerdrup et al. (2016) pointed, financial crises are rare events and have historically ocurred every 15-20 years on average,

[1] Inflation targeting doctrine is characterized by the announcement of official target ranges for the inflation rate at one or more horizons, and by explicit acknowledgment that low and stable inflation is the overring goal of monetary policy. This implies that price stability is the fundamental goal of monetary policy.

Figure 1: Macroeconomic consequences of financial crises



Notes: Data for financial crises events in emerging markets is taken from Calvo et. al. (2006). They define these episodes as events with large output collapses, large current account reversals and spikes in the EMBI spread.

so trying to prevent financial crises can potentially lead to a loss of central bank credibility. Given this difficulty, it is more practical for a central bank to decide movements in the policy interest rate after the financial crisis happens than to try to prevent it from occurring. Nonetheless, the recent financial crisis showed that monetary policy has limitations when it comes to cleaning up after a crisis occurs, and as pointed by Woodford (2012), it is not necessary to predict exactly when the crisis will occur; it suffices that the central bank identifies circumstances under which the risk of crisis increases.

A second ground of skepticism is based on doubts about the influence of monetary policy decisions in the buildup of risks to financial stability, specially when considering effects on the stock-market or real-estate "bubbles". Even granting that it might be possible to identify such risks in real time, monetary policy has major limitations. If interest-rate policy is relevant to the valuation of stocks, the change in monetary policy required to control "bubbles" in assets prices would be very severe. But again, Woodford (2012) argues that the real issue should not be one of using short-term rates in order to control possible mis-pricing of assets, but rather, one of seekeing to deter high levels of debt. Once the problem is recast in this way, the relevance of central bank decisions is more clear.

Added to this, it has also been argued that even if monetary policy is able to influence the risk of occurrence of a financial crisis, there are better tools available for this purpose. A new view has emerged suggesting that monetary policy alone cannot adequately manage the external shocks facing small emerging economies. Following the "Tinberger norm", a certain number of targets requires the policy maker to control an equal number of instruments, and in that case monetary policy should instead be supplanted by well-targeted policies, as macroprudential, regulatory, or supervisory policies (Farhi and Werning (2012)). A conflict may arise between the use of monetary policy to control risks of financial crises, and the use of it to maintain price stability and stable real activity. However, despite this tension between alternative objectives or even if monetary policy can only stabilize inflation around a given inflation target and GDP growth rate around an estimated long-run sustainable rate, it might be possible that incorporating financial stability concerns in monetary policy deliberations results in an effective strategy to deal with the problem, without costs to subordinating interest-rate policy to that end.

Motivated by the previous debate, this paper contributes to the literature analyzing the macroeconomic implications of adapting the actual framework used to structure monetary policy deliberations in a way that takes account of the possibility to prevent the occurrence of financial crises. In order to address the concerns raised above, it is essential that the ocurrence of crises is not treated as purely exogenous, as it is in the analysis such as those of Curdia and Woodford (2010), Kiyotaki et al. (2010), or Gertler and Karadi (2015), that treat the question of how central bank policy can mitigate the effect of a crisis in the event that one occurs.[2] To do so we focus on a regime-switching newkeynesian model for a small open economy with an occasionally binding financial friction that allows for endogenous financial crises.

As Mendoza (2010) and Benigno et al. (2013) showed, macroeconomic models with these kind of financial frictions have proven to be capable to describe both regular business cycle (normal times when market access is unconstrained) and crises events (when the market access is curtailed). Then, we introduce in this baseline model some sort of nominal rigidity,

[2] Those authors use a different definition of financial crises, understanding these events as situations in which there are burst in "bubbles" in assets prices.

following Calvo (1983). The importance of the role of nominal stickiness in small open economy models has been emphasized by Galí and Monacelli (2005), Benigno and Paoli (2010) and Gertler and Karadi (2011), to name a few. Monetary policy is conducted using a Taylor rule, so the monetary authority adjusts the nominal interest rate to meet an inflation rate target (and possibly, a GDP growth rate or debt growth rate target). After that, it is possible to define two regimes: one regime that describes normal economic times and full access to foreign borrowing, and other regime that describes crisis periods where the borrowing constraint binds. The scope for policy intervention in this model follows from an endogenous transition probability between regimes; that is, the switch between normal economic times and crises times is based on a time-varying probability, determined by economic variables in the economy, but the switch back to normal times (conditional on being in the crisis regime) is exogenous and constant.

The results of the model provide evidence showing that monetary policy, currently represented under the IT scheme, has major limitations when it seeks to prevent the occurrence of financial crises. Our quantitative analysis shows that as the monetary authority responds more strongly to increases in the external debt growth rate, the frequency with which the economy goes into crisis is not significantly different from the current case, in which it does not take into account this variable in its policy decision. However, the volatility of inflation is much higher, which may put in danger the fundamental purpose of IT: keeping inflation expectations anchored to achieve sustainable price stability. The volatility of consumption also increases, which goes against the willingness of households to smooth consumption. On the other hand, there is a decreasing trend in the occurrence of financial crises when the central bank is more concerned about the GDP growth gap, although the magnitude of the reduction is small. The volatility of inflation and consumption also increases, although in a much smaller proportion relative to the case in which the external debt growth rate is targeted. These results tend to support the idea that, although the monetary policy can contribute to preventing financial crises, changes required to make a significant difference would be very severe.

This rest of the paper is organized as follows. Section 2 presents the baseline New Keynesian DSGE model. Section 3 describes the regime-switching approach, while Section 4 provides a description for the calibration procedure. In Section 5 we discuss the solution method and performance. Section 6 assesses the macroeconomic consequences of adopting alternative Taylor rules. The last section concludes.

2 The Model

The model is a regime-switching dynamic stochastic general equilibrium model (RS-DSGE) that describes a small open economy, populated by households, firms and the central bank. In particular, it adds the possibility of financial crises in a two sector New Keynesian model. The basic setup is as follows. Households supply labor to firms and consume final goods, save in form of domestic bonds, adquire foreign debt and receive the revenues from firms. The structure of production is distributed in two sectors: tradable and nontradable. Firms in the tradable sector produce in a competitive environment an homogeneous good that can be traded internationally, and in that sense, can be consumed by domestic or foreign households. Nontradable output is produced in several stages in a monopolistic competitive enviroment with nominal rigidities. Financial markets are not only incomplete but also imperfect as in Mendoza (2010) and Benigno et al. (2013). The financial friction comes from an occasionally binding credit constraint which states that households debt stock cannot be greater than a fraction of their current total income. This allows to define a regime of normal times and another one of crisis times. We also consider a monetary authority who conduct monetary policy following a Taylor rule.

2.1 Households

There is a representative household that maximize the expected present discounted value of utility:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\rho} \left(z_t^u C_t - \frac{H_t^\delta}{\delta} \right)^{1-\rho} \right] \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator with information up to period $t = 0$, $\beta \in (0, 1)$ is the intertemporal discount factor, $\rho > 1$ is the constant relative risk aversion coefficient and $\delta > 1$ is the inverse of the Frisch elasticity of the labor supply. Here, C_t denotes the consumption basket and H_t the supply of labor for the tradable and nontradable sectors ($H_t = H_t^T + H_t^N$). The assumption of perfect sustituibility between labor services in the two sectors ensures that there is an unique labor market. In (1), the preference specification follows Greenwood et al. (1988) in which utility is defined in terms of the excess of consumption relative to the disutility of labor. This functional form eliminates the wealth effect of labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. z_t^u corresponds to a preference shock, which is assumed to follow and AR(1) process:

$$z_t^u = \rho^{z^u} z_{t-1}^u + \left(1 - \rho^{z^u}\right) \log\left(\bar{z^u}\right) + \sigma^{z^u} \epsilon_t^{z^u} \tag{2}$$

with $\epsilon_t^{z^u}$ a non-serially correlated preference shock with σ^{z^u} as its standard deviation, $\rho^{z^u} \in (0,1)$ the persistence parameter and $\bar{z^u}$ the steady state value. The consumption basket is assumed to be a Cobb-Douglas compound of tradable and non-tradable goods:

$$C_t = \left(C_t^T\right)^{\omega} \left(C_t^N\right)^{1-\omega} \tag{3}$$

where C_t^T and C_t^N denote tradable and nontradable consumption goods, and $\omega \in (0,1)$ is a parameter that determines the share of tradable goods in total consumption. Regardless the level of C_t the household decides on, it will always be optimal to purchase the combination of tradable and nontradable goods that minimize the cost of achieving this level of the composite goods. In this case, the consumer price index (CPI) that corresponds to the previous specification is given by:

$$P_t = \omega^{-\omega}(1-\omega)^{-(1-\omega)} \left(P_t^T\right)^{\omega} \left(P_t^N\right)^{1-\omega} \tag{4}$$

where all prices are for goods sold in the home country, in domestic currency and at the consumer level. Taking the price of the final good as numeraire, and denoting relative prices with lowercase letters, the demand for tradable and nontradable goods, and the CPI are given by:

$$C_t^T = \frac{\omega C_t}{p_t^T} \tag{5}$$

$$C_t^N = \frac{(1-\omega) C_t}{p_t^N} \tag{6}$$

$$1 = \omega^{-\omega}(1-\omega)^{-(1-\omega)} \left(p_t^T\right)^{\omega} \left(p_t^N\right)^{1-\omega} \tag{7}$$

On the other hand, households maximize (1) subject to the budget constraint and a foreign borrowing constraint. The budget constraint in real terms is defined as:

$$w_t H_t + e_t b_t^{\star} + (1 + r_{t-1}) b_{t-1} + \Pi_t = C_t + (1 + r_{t-1}^{\star}) e_t b_{t-1}^{\star} + b_t \tag{8}$$

where w_t is the real wage, $b_t^{\star}$ is the real external debt expressed in terms of foreign consumption basket at the end of period t, b_t is a real state-contingent domestic bond at the end of period t, e_t is the real exchange rate (measured in units of domestic goods in terms of foreign goods), r_t is the real interest rate of the domestic bond and $r_t^\star$ is the real interest rate that agents face in international financial markets. Households receive profits Π_t from the nontradable production sector, respectively.

Since international financial markets are incomplete and access to them is imperfect as well, there is a financial friction in the sense that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$e_t b_t^\star \leq \kappa \left(w_t H_t + \Pi_t \right) \tag{9}$$

This constraint is not derived from an optimal credit contract but imposed directly as in other macro models with endogenous credit constraints (e.g. Kiyotaki and Moore (1997), Aiyagari and Gertler (1998), or Mendoza (2010)). However, it is possible to interpret this contract as the outcome of the interaction between lenders and borrowers in which lenders are not willing to permit borrowing beyond a certain limit. This limit depends on the parameter κ that measures the tightness of the credit constraint.

Furthermore, this friction adds two important elements to the business cycle transmission mechanism that determine model's quantitative results:

1. The constraint is occasionally binding; that is, it only binds when the external debt is sufficiently high. Tipical realizations of exogenous shocks that increase external debt, or decrease total income, produce financial crises when the state of the economy is such that external debt is near the borrowing limit. If the constraint does not bind, the shocks yield similar macroeconomic responses as in a typical small open economy model with nominal rigidities. As a result, the economy displays "normal" business cycle patterns when the credit constraint does not bind.

2. The loss of credit market is endogenous. In particular, high levels of external debt at which the credit constraint binds are reached because realizations of exogenous shocks lead the endogenous business cycle dynamics to states with sufficiently high debt.

Following Mendoza (1991), it is important to note that it is not possible to produce financial crises using standard DSGE models, since agents still have access to frictionless credit markets, and because of that negative productivity shocks induce typical RBC-like responses. Large shocks could trigger large output collapses but this would still fail to explain the current account reversal and the collapse in consumption, given that households would borrow from abroad to smooth consumption.

The external interest rate is modeled as in Schmitt-Grohe and Uribe (2003), having two components: one, the international risk free interest rate, and second, a risk component, which is a positive function of the deviations of the external debt to GDP with respect to its steady state value. That is:

$$r_t^\star = r_t^f + \Phi\left[\exp\left(\frac{e_t b_t^\star}{Y_t} - \Upsilon\right) - 1\right] \tag{10}$$

where $\Phi > 0$ is a parameter that determines the elasticity of the risk component to deviations of the external debt to GDP ratio from its steady state, Υ is the steady state value of the external debt to GDP and r_t^f corresponds to the international risk free real interest rate, which is assumed to follow and AR(1) process:

$$r_t^f = \rho^{rf} r_{t-1}^f + \left(1 - \rho^{rf}\right)\log\left(\bar{rf}\right) + \sigma^{rf}\epsilon_t^{rf} \tag{11}$$

with ϵ_t^{rf} a non-serially correlated risk free interest rate shock with σ^{rf} as its standard deviation, $\rho^{rf} \in (0,1)$ the persistence parameter and $\bar{rf}$ the steady state value. Given that, household choose $\{C_t,\, H_t,\, b_t^\star,\, b_t\}_{t=0}^\infty$ to maximize (1) subject to (8) and (9). The first-order conditions of this problem over consumption, labor, foreign debt and domestic bonds are the following:

$$\left(z_t^u C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} = \lambda_t \tag{12}$$

$$\left(z_t^u C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} H_t^{\delta-1} = \lambda_t w_t + \kappa\mu_t w_t \tag{13}$$

$$e_t\lambda_t = \beta\mathbb{E}_t(1 + r_t^\star)e_{t+1}\lambda_{t+1} + e_t\mu_t \tag{14}$$

$$\lambda_t = \beta\mathbb{E}_t(1 + r_t)\lambda_{t+1} \tag{15}$$

where λ_t is the multiplier on the period budget constraint and μ_t is the multiplier on the international borrowing constraint. Similar to Benigno et al. (2013), when the credit constraint is binding ($\mu_t > 0$), Euler equation (14) incorporates an effect related to the risk on external financing. The left hand side measures the benefits of increasing indebtedness this period (depleting net foreign assets) in terms of the foreign good. The right hand side measures the costs which are two-fold. First, there is a decrease in net wealth brought into the next period, which decrease utility through a decrease in consumption. Second, there is the cost to households in terms of tightening the current borrowing constraint, measured by the second expression on the right hand side. Even when the constraint is not binding at time t, there is an intertemporal effect coming from the possibility that the constraint might be binding at time $t+1$, which is represented by the term $\mathbb{E}_t\mu_{t+1}$.[3] This means that current total consumption would be lower compared to an economy in which access to foreign borrowing is unconstrained.

Furthermore, when combining (12) and (13) the labor supply schedule is:

$$H_t^{\delta-1} = \left(1 + \kappa\frac{\mu_t}{\lambda_t}\right) w_t \tag{16}$$

Note here that when the credit constraint is binding the marginal utility of supplying one more unit of labor is higher, and this helps to relax the constraint. In consequence, labor supply becomes steeper and agents substitute leisure with labor in order to increase the value of GDP, because it acts as a "collateral" in (9). So, for given wages and prices, households prefer to work more when access to foreign financial markets is curtailed.

Now, combining (14) and (15):

$$\left(\frac{1+r_t^\star}{1+r_t}\right)\frac{\mathbb{E}_t e_{t+1}}{e_t} + \frac{\mu_t}{\lambda_t} = 1 \tag{17}$$

[3] Note that if the borrowing constraint does not bind at time t, the Euler equation becomes:

$$e_t\lambda_t = \beta\mathbb{E}_t(1+r_t^\star)e_{t+1}\lambda_{t+1}$$

Now, taking (14) one period ahead, and including it in the last expression, we obtain:

$$e_t\lambda_t = \beta\mathbb{E}_t(1+r_t^\star)\left[\beta\mathbb{E}_{t+1}(1+r_{t+1}^\star)e_{t+2}\lambda_{t+2} + e_{t+1}\mu_{t+1}\right]$$

$$e_t\lambda_t = \beta^2\mathbb{E}_t(1+r_t^\star)(1+r_{t+1}^\star)e_{t+2}\lambda_{t+2} + \beta\mathbb{E}_t(1+r_t^\star)e_{t+1}\mu_{t+1}$$

Note that when there is not any restriction in capital flows, the standard result of uncovered interest parity (UIP) is obtained: the rate of return on the domestic bond is equal to the expected rate of return on the foreign debt (which is adjusted by the expected depreciation). Now, lack of free capital mobility implies that the rate of return of the domestic bond is greater than the expected rate of return on the foreign debt.

2.2 Firms

There are two sectors in this economy: (i) a tradable sector, composed by a firm that produces an homogeneous good that can be traded internationally, and in that sense, can be consumed by domestic or foreign households, and (ii) a nontradable sector, composed by a firm that produces a final nontradable good for consumption exclusively by domestic households, using production of intermediate firms as inputs. These intermediate firms produce nontradable differenciated goods using a constant returns to scale technology. Given that a key feature of a New Keynesian model is its assumption that there are price-setting frictions, the presence of these frictions requires that intermediate firms have the power to set prices, and this in turn requires monopoly power. The Dixit and Stiglitz (1977) framework of production create an enviroment in which there is monopoly power without contradicting the fact that actual economies have a very large number of firms, since it is assumed that there is a continuum of price-setting monopolist firms of measure 1.

2.2.1 Tradable sector

There is a representative firm that produces a homogeneous tradable good with a variable labor input and constant returns to scale technology. The available technology is represented by the following production function:

$$Y_t^T = z_t^T H_t^T \tag{18}$$

where Y_t^T is the tradable good production and z_t^T is the tradable sector productivity that follows an AR(1) process:

$$z_t^T = \rho^{z^T} z_{t-1}^T + \left(1 - \rho^{z^T}\right) \log\left(\bar{z^T}\right) + \sigma^{z^T} \epsilon_t^{z^T}$$

with $\epsilon_t^{z^T}$ a non-serially correlated productivity shock with σ^{z^T} as its standard deviation, $\rho^{z^T} \in (0,1)$ the persistence parameter and $\bar{z^T}$ the steady state value. The labor demand consistent with the profit maximization problem is given by:

$$w_t = p_t^T z_t^T \tag{19}$$

so that the value of the marginal product of labor in tradable sector equals real wage. Since there are flexible prices in this sector, Purchasing Power Parity (PPP) holds for tradable goods:

$$p_t^T = e_t p_t^{T\star}$$

where $p_t^{T\star}$ is the real price of tradable goods in terms of foreign consumer price index. The latter evolves according to an exogenous AR(1) process:

$$p_t^{T\star} = \rho^{p^{T\star}} p_{t-1}^{T\star} + \left(1 - \rho^{p^{T\star}}\right) \log\left(\bar{p^{T\star}}\right) + \sigma^{p^{T\star}} \epsilon^{p^{T\star}}$$

with $\epsilon^{p^{T\star}}$ a non-serially correlated productivity shock with $\sigma^{p^{T\star}}$ as its standard deviation, $\rho^{p^{T\star}} \in (0,1)$ the persistence parameter and $\bar{p^{T\star}}$ the steady state value.

2.2.2 Nontradable sector

A homogeneous nontradable good, Y_t^N, is produced by a representative, competitive firm using the following technology:

$$Y_t^N = \left(\int_0^1 Y_t^N(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} \tag{20}$$

where θ governs the elasticity of substitution between different inputs. The representative firm takes the real price of nontradable output, p_t^N, and the price of inputs, $p_t^N(j)$, as given. Profit maximization leads to the following first order condition:

$$Y_t^N(j) = p_t^N(j)^{-\theta} Y_t^N \tag{21}$$

Substituting (21) into (20) yields the following relation between the aggregate relative price level and the relative prices of intermediate goods:

$$p_t^N = \left[\int_0^1 p_t^N(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{22}$$

On the other hand, the j-th intermediate good firm produce using a variable labor input and constant returns to scale technology. The available technology is represented by the following production function:

$$Y_t^N(j) = z_t^N H_t^N(j) \tag{23}$$

where $H_t^N(j)$ denotes labor services at nontradable sector hired by the firm, and z_t^N is the productivity that follows an AR(1) process:

$$z_t^N = \rho^{z^N} z_{t-1}^N + \left(1 - \rho^{z^N}\right) \log\left(\bar{z^N}\right) + \sigma^{z^N} \epsilon_t^{z^N}$$

with $\epsilon_t^{z^N}$ a non-serially correlated productivity shock with σ^{z^N} as its standard deviation, $\rho^{z^N} \in (0,1)$ the persistence parameter and $\bar{z^N}$ the steady state value. From the cost minimization problem we obtain the firm's real marginal cost, in terms of nontradable goods, denoted by φ_t:

$$\varphi_t = \frac{w_t}{p_t^N z_t^N} \tag{24}$$

Additionally, j-th firm is a monopolistic in the production of the j-th good, and so it set its price. Profit maximization is subject to two constraints: The first is the demand curve each firm faces. This is given by (21). The second constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness is due to Calvo (1983). Here, a fraction $(1-\epsilon)$ of firms, which are denoted as "forward-looking" set prices optimally using all the available information in order to forecast future marginal costs. The remaining firms, of measure ϵ, which are refered to as "backward looking", instead use a simple rule of thumb which requires only some past information on aggregate nontradable price behavior. Whenever the firm is not allowed to reset its price, it is automatically increased at the gross inflation rate of the previous period. That is:

$$P_{t+i}^N(j) = P_t^N(j) \left(\prod_{s=1}^{s=i} \left(1 + \pi_{t+s-1}^N\right)\right) \tag{25}$$

where $(1 + \pi_t^N)$ is the gross inflation rate at period t. Note that the parameter ϵ is a measure of the degree of nominal rigidity: a larger ϵ implies that fewer firms re-optimize each period and that the expected time between price changes optimally is longer. Therefore, intermediate firm's price setting problem is summarized as follows:

$$\max_{p_t^N(j)} \Pi_t = \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \Delta_{t,t+i} \left[p_{t+i}^N(j) Y_{t+i}^N(j) - \varphi_{t+i} Y_{t+i}(j)\right]$$

subject to (21) and (25). Here, $\Delta_{t,t+i} = \beta^i \frac{\lambda_{t+i}}{\lambda_t}$ is the household's stochastic discount factor, since it is the owner of the firms. Let $\hat{p_t^N}(j) \equiv \frac{P_t^N(j)}{P_t^N}$ be the optimal price, in terms of nontradable goods, chosen by a firm adjusting at time t. The first-order condition associated to the latter problem is (see Appendix A):

$$\hat{p_t^N}(j) = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} \left(\frac{1+\pi_{t+i}^N}{1+\pi_t^N}\right)^{\theta} Y_{t+i}^N}{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \left(\frac{1+\pi_{t+i}^N}{1+\pi_t^N}\right)^{\theta-1} Y_{t+i}^N} \tag{26}$$

Note that (26) has a recursive expression given by:

$$\hat{p_t^N}(j) = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbb{E}_t \Theta_t}{\mathbb{E}_t \Psi_t} \tag{27}$$

where:

$$\Theta_t = \lambda_t \varphi_t Y_t^N + \epsilon\beta \left(\frac{1+\pi_{t+1}^N}{1+\pi_t^N}\right)^{\theta} \Theta_{t+1} \tag{28}$$

$$\Psi_t = \lambda_t Y_t^N + \epsilon\beta \left(\frac{1+\pi_{t+1}^N}{1+\pi_t^N}\right)^{\theta-1} \Psi_{t+1} \tag{29}$$

While these firms produce differenciated products, they all have the same production technology, and face demand curves with constant and equal demand elasticities. Because of this, all firms adjusting in period t will set the same price. Thus, the price index can be expressed as:

$$1 = \epsilon \left(\frac{1+\pi_{t-1}^N}{1+\pi_t^N}\right)^{1-\theta} + (1-\epsilon)\hat{p_t^N}(j)^{1-\theta} \tag{30}$$

which is the conventional Calvo-pricing equation for the determination of prices.

2.3 Central Bank

Next, we turn to the description of the central bank. Since it is assumed that the economy has sticky prices, there is a role for monetary policy. The central bank is assumed to set the short term nominal interest rate i_t according to the following rule:

$$i_t = \rho^i i_{t-1} + (1-\rho^i)\left[\bar{r} + \bar{\pi} + \Omega_\pi(\pi_t - \bar{\pi}) + \Omega_y(y_t - \bar{y}) + \Omega_b(\hat{b_t^\star} - \bar{\hat{b}^\star})\right] + \sigma^i \epsilon_t^i \tag{31}$$

where $\rho^i \in (0,1)$ is the degree of nominal interest rate smoothing, $\bar{\pi}$ is a fixed inflation target, $\bar{r}$ is the steady state of the real interest rate, y_t is the net GDP growth rate, $\hat{b_t^\star}$ is the external debt growth rate, $\bar{\hat{b^\star}}$ is the steady state value of the external debt growth rate, $\bar{y}$ is the steady state value of the net GDP growth rate (which in this case are both equal to zero), Ω_π, Ω_y and Ω_b are the degree of responsiveness of the central bank to deviations of inflation from its target, to deviations of net GDP growth rate from its steady state value and to deviations of the external debt growth debt from its steady state value, respectively. ϵ_t^i is an *i.i.d.* monetary policy shock with standard deviation σ^i. Note that the gap in the external debt growth rate represents the nonconventional component of monetary policy.

2.4 Market clearing conditions

Using the household's budget constraint, it can be shown that, by using the equilibrium condition for the domestic bond market, $b_t = 0$, the balance of payments of this economy is:

$$Y_t = C_t + (1 + r_{t-1}^\star)e_t b_{t-1}^\star - e_t b_t^\star \tag{32}$$

Finally, using the definition of firm profits and wages, the credit constraint implies that the amount that the agents can borrow is limited by a fraction of the value of its GDP:

$$e_t b_t^\star \leq \kappa Y_t \tag{33}$$

So that (32) and (33) determines the evolution of foreign borrowing.

3 Regime-Switching

We characterize the possibility of a financial crisis in the model through regime switchiong. It is assumed that there are two regimes: one that represents "normal times", in which the constraint does not bind, characterized by full access to international financial markets, high discount factor and high productivity; and one of "crises times", in which the constraint binds, characterized by curtailed access to foreign borrowing, low discount factor and low productivity. The assumption that economy jumps between two discrete states is obviously an oversimplification, but if captures something important about financial crises of the kind with which we are concerned: that they are typically characterized by sudden reversal in international capital flows and substantial decreases in absortion and production that are instead relatively stable under normal circumstances.

In addition to this, Mendoza (2010) provide some evidence on falls in total factor productivity and "impatience effects" related to a reduction in the intertemporal discount factor during financial crisis situations. This suggests that in times of restricted access to financial markets agents are more concerned about their current state, relative to their future state. A regime-switching model is a parsimoniously parameterized way of capturing those periods of crises. An advantage of this approach is that it responds to a common complaint about policy analyses using standard DSGE models, namely, that local approximation abstracts from the possibility of occasional excursions far from the the normal range of variation in the state variables as a result of nonlinearities, which extreme outcomes are precisely the ones that one must be concerned about in an analysis of risks to financial stability. A regime-switching model allows for a non-trivial probability of occasional excursions far from normal conditions, and allows the probability of such excursions to be endogenous.

Based on the previous description, we define the variable $s_t \in \{0, 1\}$ that indicates the regime, and follows a two-state Markov chain: if the credit constraint binds, $s_t = 0$, if it does not bind, $s_t = 1$. Notice that it is possible to express the complementary slackness condition given by (33) as:

$$\mu_t \left(\frac{-e_t b_t^\star}{Y_t} + \kappa \right) = 0$$

with $\mu_t \geq 0$ and $\frac{-e_t b_t^\star}{Y_t} + \kappa \geq 0$. Note that $\frac{-e_t b_t^\star}{Y_t} + \kappa$ represents the distance between the external debt to GDP ratio and the limit imposed in the international financial markets. Therefore, if the constraint binds, $\frac{-e_t b_t^\star}{Y_t} + \kappa = 0$ and $\mu_t > 0$, and when it does not, $\frac{-e_t b_t^\star}{Y_t} + \kappa > 0$ and $\mu_t = 0$. This regime-switching variable s_t turn "on" and "off" the relevant portions of the slackness condition. The functional form assumed here is:

$$s_t \mu_t + (1 - s_t) \left(\frac{-e_t b_t^\star}{Y_t} + \kappa \right) = 0 \tag{34}$$

First, note that when $s_t = 1$, equation (34) simplifies to:

$$\mu_t = 0$$

which means that the borrowing constraint is not binding, and the the economy displays "normal" business cycle as in a typical small open economy model with nominal rigidities. Also, when $s_t = 0$, equation simplifies to:

$$\frac{-e_t b_t^\star}{Y_t} + \kappa = 0$$

which implies that the access to external credit is curtailed, and the level of debt is limited by a fraction of the GDP. In addition to the credit constraint and consistent with Mendoza (2010), steady state value for productivities and discount factor change at the same time as the regime. In particular, the laws of motion switch intercepts, so $\beta = \beta(s_t)$ and $\bar{z^i} = z^i\bar{(}s_t)$ for $i \in \{T, N\}$. Note that this implies that the steady state real interest rate also takes different values depending on the regime.

3.1 Endogenous transition probabilities

As mentioned before, endogenous transition probabilities allows to analyze the relationship between monetary policy and prevention of financial crises, since decisions about short-term interest rate can potencially reduce the probability that economy will come in foreign financial difficulties. Specifically, when the credit constraint is not binding, the probability that it binds the next period depend on the value of the external debt relative to the credit limit. In particular, the funtional form for this probability is:

$$\Pr\left(s_{t+1} = 0 \mid s_t = 1\right) = \frac{\exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t} + \kappa\right)\right]}{\zeta + \exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t} + \kappa\right)\right]}$$

where γ is a parameter capturing the sensibility of the probability to the distance between the current level of debt to GDP and its limit, and ζ is a small constant, close to zero. When fixing a value for ζ sufficiently close to zero, and a value for γ sufficiently high (tending to infinity), the probability achieve the threshold behavior of the original constraint: if the distance between the debt and its limit is very close to zero, the probability that becomes binding next period tends rapidly to one; but if this distance is positive and sufficiently far from zero, the probability of reaching the other regime is zero. Furthermore, when the constraint is binding, the probability that it does not bind the next period is constant and given by:

$$\Pr\left(s_{t+1} = 1 \mid s_t = 0\right) = \varpi$$

Then, the transition matrix is:

$$\mathbf{P} = \begin{bmatrix} P_{00,t} & P_{01,t} \\ P_{10,t} & P_{11,t} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t}+\kappa\right)\right]}{\zeta+\exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t}+\kappa\right)\right]} & \frac{\exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t}+\kappa\right)\right]}{\zeta+\exp\left[-\gamma\left(\frac{-e_t b_t^\star}{Y_t}+\kappa\right)\right]} \\ \varpi & 1-\varpi \end{bmatrix}$$

where $P_{ij,t} = \Pr(s_{t+1} = i \mid s_t = j)$.

3.2 Competitive Equilibrium

To characterize the competitive equilibrium, the following definitions are used:

Definition 1. A price system is a positive sequence $\{w_t, r_t, r_t^\star, p_t^N, p_t^T, \hat{p_t^N}, e_{t,} i_t\}_{t=0}^\infty$.

Definition 2. $\{r_t^f, p_t^{T\star}, z_t^T, z_t^N, z_t^u\}_{t=0}^\infty$ are taken as exogenous sequences. $b_0, b_0^\star > 0$ are also taken as given. An equilibrium is a price system, a sequence of consumption $\{C_t^T\}_{t=0}^\infty$, $\{C_t^N\}_{t=0}^\infty$ and $\{C_t\}_{t=0}^\infty$, number of hours worked $\{H_t^T\}_{t=0}^\infty$,$\{H_t^N\}_{t=0}^\infty$ and $\{H_t\}_{t=0}^\infty$, domestic real private debt $\{b_t\}_{t=0}^\infty$, external real debt $\{b_t^\star\}_{t=0}^\infty$, tradable and non-tradable production $\{Y_t^T\}_{t=0}^\infty$, $\{Y_t^N\}_{t=0}^\infty$ in order that:

1. Given the price system and the firm's profits transfers, household's optimal control problem is solved with $\{H_t^T\}_{t=0}^\infty$, $\{H_t^N\}_{t=0}^\infty$, $\{H_t\}_{t=0}^\infty$, $\{C_t^T\}_{t=0}^\infty$, $\{C_t^N\}_{t=0}^\infty$, $\{C_t\}_{t=0}^\infty$, $\{b_t = 0\}_{t=0}^\infty$ and a level of $\{b_t^\star\}_{t=0}^\infty$ such that $H_t = H_t^T + H_t^N$ is satisfied.
2. Given the price system, tradable firm's optimal control problem is solved with $\{H_t^T\}_{t=0}^\infty$ and $\{Y_t^T\}_{t=0}^\infty$, and non-tradable firm's optimal control problem is solved with $\{H_t^N\}_{t=0}^\infty$ and $\{Y_t^N\}_{t=0}^\infty$.
3. The policy rule (31), the borrowing constraint (33) and the balance of payments (32) are satisfied for all $t > 0$.

The complete set of model equations is summarized at Appendix B.

4 Calibration

Despite a significant body of empirical work on identifying financial crises in emerging markets which seeks to describe the macroeconomic dynamics around this events, there is no consensus on how to define financial crises empirically. In order to contrast data with model outcomes, and consistent with the model and the empirical literature, we follow a definition based on Calvo et al. (2006). In particular, a financial crisis is an event composed by the union of: (i.) a GDP window, containing a large fall in GDP growth rate exceeding one standard deviation from its mean (that starts when the fall in the growth rate of GDP exceeds one standard deviation, and ends when is smaller than one standard deviation) that overlaps at any point in time with a (ii.) capital flow window, that contains a fall in capital flows exceeding one standard deviation from its mean, and (iii.) an aggregate spread window, containing a spike in the aggregate EMBI spread exceeding one standard deviation from its mean.

This methodology applied to Colombian data allows us to identify a crisis event for the period 1998Q4:1999Q3. In this paper we focus in Colombia for several reasons. First, Colombia experienced an episode of international capital flow reversal that is unambiguously recognized as an example of financial crisis: The "End of Century" financial crisis of 1998-1999, which in turn was characterized by a large contraction in GDP and consumption, a fall in the real wage index and a hike in real interest rate and spread. Second, Colombia is a good example of a market-based small open economy in which production in both the tradable and nontradable sectors is an important source of business cycle fluctuations. Third, the central bank addopted IT in 1991, and has repeteadly expressed its interest in contribuiting to close the current account deficit and avoid macroeconomic imbalances. In the monetary policy minutes they usually mention that "the Board will continue monitoring expenditure adjustment and its consistency with the long-term income level, the sustainability of the external deficit and, in general, macroeconomic stability". Table 1 reports the structural and monetary policy parameters values of the model. Data frequency is quarterly for the period 1994Q1-2016Q1.

First, we expose the values of the estructural parameters. Estimates of the elasticity of intertemporal substitution are uncertain, but this parameter is set to 4 as in Hamann et al. (2015) and González et al. (2011). The wage elasticity of labor supply is set to 2 as in Arellano and Mendoza (2002) and Benigno et al. (2013). The tradable consumption

Table 1: Model parameters using colombian data: 1994Q1-2016Q1

Structural parameters	**Values**
Intertemporal substituion and risk aversion	$\rho = 4$
Labor supply elasticity	$\delta = 2$
Relative weight of tradable and nontradable goods	$\omega = 0.4$
Discount factor under normal times	$\beta(0) = 0.9914$
Discount factor under crises times	$\beta(1) = 0.9714$
Credit constraint parameter	$\kappa = 1.69$
Sensibility of the crises probability to debt limit	$\gamma = 5000$
Probability of exit from crises	$\varpi = 0.98$
Constant in the probability of going to crises	$\zeta = 0.0001$

Monetary policy parameters	**Values**
Degree of price stickiness	$\epsilon = 0.75$
Inflation target	$\bar{\pi} = 0.0074$
Degree of competition in the differentiated market	$\theta = 6$
Degree of smoothness in the policy rule	$\rho^i = 0.8$
Weight to inflation gap	$\Omega_\pi = 2.5$

share in total consumption is 0.4, according to data from the Colombian national statistics department, DANE. β, which in steady state is equal to $\frac{1}{1+r}$ is fixed at 0.9914 under normal times, according to González et al. (2011) who set the annual long term real interest rate for Colombia in 3.5 %, which corresponds to 0.86 % quarterly. Under crises times, β switchs to 0.9714 in order to match an annual real interest rate of 12.3 % observed in the colombian financial crisis, which implies a 2.94% quarterly. Furthermore, by setting the credit constraint parameter, κ, in 1.692 we obtain the level of external debt to GDP observed in the "End of Century" financial crisis.

Now, considering the monetary policy parameters, the parameter θ that determines the degree of competition in the differentiated nontradable goods market, is set to 6 since this value is assumed in several models in Colombia. The degree of nominal interest rate smoothing is equal to 0.8, which is close to the value considered in González et al. (2011). The

Table 2: Model parameters using colombian data: 1968-2014

Exogenous variables	**Values**
External debt to GDP under normal times	$\Upsilon = 1.12$
Risk-free real interest rate	$r^f = 0.0086$
Elasticity of risk component	$\Phi = 0.000742$
Tradable productivity under normal times	$z^T(0) = 4$
Tradable productivity under crises times	$z^T(1) = 3.1$
Nontradable productivity under normal times	$z^N(0) = 4$
Nontradable productivity under crises times	$z^N(1) = 3.1$
Persistence parameters	**Values**
Persistence of tradable productivity	$\rho^{z^T} = 0.83$
Persistence of nontradable productivity	$\rho^{z^N} = 0.83$
Persistence of risk-free interest rate shock	$\rho^{r^f} = 0.91$
Persistence of external price of tradable goods	$\rho^{p^{T\star}} = 0.8$
Persistence of preference shock	$\rho^{z^u} = 0.9$
Standard deviations	**Values**
Standard deviation of tradable productivity	$\sigma^{z^T} = 0.068$
Standard deviation of nontradable productivity	$\sigma^{z^N} = 0.07$
Standard deviation of risk-free interest rate shock	$\sigma^{r^f} = 0.07$
Standard deviation of external price of tradable goods	$\sigma^{p^{T\star}} = 0.051$
Standard deviation of preference shock	$\sigma^{z^u} = 0.189$
Standard deviation of monetary policy shock	$\sigma^i = 0.073$

parameterϵ that determines the degree of price stickiness is set to 0.75, in order to have nontradable prices changing once every year, based on the estimation obtained by Bejarano (2005) for Colombia. The inflation target $\bar{\pi}$ is fixed among regimes at 3 % (annual rate) according to the target set by the central bank, corresponding to 0.7417 % quarterly. Notice that the long run nominal interest rate is fixed according to r and $\bar{\pi}$, and it takes different values in each regime. The parameter Ω_π corresponding to the weight given by the monetary authority to the inflation gap is in 2.5, based on Bonaldi et al. (2011).

The parameter Υ is fixed at 1.15, and in order to pin down the external debt to GDP under normal times. The long run risk-free interest rate, $\bar{r^f}$, is equal to $\bar{r}$ under normal times. Following Schmitt-Grohe and Uribe (2003), the elasticity of the risk component to deviations of the external debt to GDP ratio from its steady state, Φ, takes the value of 0.000742. On the other hand, we assume that the steady state values of the productivities in normal times are 4, and in crises times these fall to 3.1. This is so in order to calibrate the fall in the real wage observed in the 1998-1999 financial crisis event. The parameters γ and ζ are 5000 and 0.0001 in order to achieve the threshold behavior of the original constraint, and ϖ is set to 0.98 as we seek to match the fraction of time that the economy is in crisis.

The persistence of the productivity processes of both sectors is set to 0.83, according with the Solow residual estimation for Colombia (Restrepo (2005)). The standard deviations are set in 0.068 and 0.07 for the tradable and nontradable production sector, respectively, in order to reproduce as close as possible the observed GDP volatility. The persistence coefficient of the risk-free interest rate is set to 0.91 according to an OLS estimation for the LIBOR 3M autocorrelation coefficient of the cyclical component. For the external price of tradable goods, persistence takes the value of 0.8 according to the estimated autocorrelation of the external CPI. Finally, we assume a value for ρ^{z^u} of 0.9 is assumed according to Restrepo (2005) estimation. The standard deviations for the risk-free rate shock, the external price of the tradable goods and the preference shock are assumed to 0.07, 0.051 and 0.189, respectively, in order to replicate the consumption and the current account volatilities that are observed in the data. The standard deviation of the monetary policy shock is set at 0.073 according to the cyclical component of Colombia's interbank rate.

5 Model Dynamics

5.1 Solving the Model

The characterization of the equilibrium of the model presented in Section 2 does not yield a closed form solution. Moreover, given the regime-switching structure of the model, standard solution techniques based on approximation can not be applied. In the recent literature on regime-switching DSGE models, various alternative techniques of solving those models has been suggested (Farmer et al. (2011), Svensson and Williams (2009), Maih (2015), etc.). The technique we use is that of Maih (2015). There are three reasons of using this method over the others:

1. This method is perturbation based and it can handle many state variables, which are usually present in monetary policy models; this implies that there is not "curse of dimensionality" problems, and it is possible to analyze the model dynamics considering multiple exogenous shocks.

2. It allows for endogenous transition probabilities, while the others do not. As mentioned before, this endogeneity of the probability of ocurrence of a crisis raises the question of the implications of monetary policy decisions in prevention of financial crises.

3. When the underlying structural model is nonlinear (as in this case), the agents are aware of the nonlinear nature of the system and of the switching process. Other solution approaches start out with a linear model and then apply regime-switching to the parameters. This has implications for the solutions based on approximation and for the decision rules.

Denoting $\mathbf{Y}$ as the vector composed by control, co-state and flow variables, the solution of the model is of the form:

$$
\begin{aligned}
\mathbf{Y}_t &= \mathbf{H}(s_t)\mathbf{x}_t \\
\mathbf{x}_{t+1} &= \mathbf{M}(s_t)\mathbf{x}_t + \mathbf{R}\mathbf{e}_{t+1}
\end{aligned}
\tag{35}
$$

where $\mathbf{x}$ is a vector of endogenous and exogenous states, s represents the regime, $\mathbf{H}$ characterizes the policy function and $\mathbf{M}$ the state transition matrix. $\mathbf{e}_{t+1}$ is an innovation vector and $\mathbf{R}$ is a matrix composed of zeros or ones. This matrix determines which variables are hit by the shock and in what magnitude. Notice that $\mathbf{H}$ and $\mathbf{M}$ depend on the regime, given that agents behave differently under crisis times and normal times. Given that there is not analytical solution to this kind of models, this method relies on a perturbation that allow to approximate the decision rules in (35).

5.2 Simulation

This class of model is capable of describing well both the business cycle facts and the dynamics around a typical financial crisis event of an emerging market economy like Colombia.

In order to contrast data with model outcomes during financial crisis episodes, consistent with the model and the empirical literature above, a financial crisis in the model is an event in which: (i) $\mu_t > 0$, i.e., the foreign borrowing constraint is binding, (ii) $b^{\star}_{t-1} - b^{\star}_t > \sigma(b^{\star}_{t-1} - b^{\star}_t)$, i.e., the current account or changes in the net foreign assets position in a given period exceed by one its standard deviation, and (iii) $Y_t < \bar{Y} - \sigma(Y_t)$, i.e., GDP in a given period is lower that its mean minus one times its standard deviation. As in Benigno et al. (2013) the first criterion is a purely model-definition of financial crisis. The second criterion allows to consider only events in which there are current account reversals, and the third criterion restricts events to situations with abrupt declines in GDP.

In the implementation of the model we use all the sources of uncertainty $\{\epsilon^{z^U}, \epsilon^{z^T}, \epsilon^{z^N}, \epsilon^{r^f}, \epsilon^{p^{T\star}}, \epsilon^i_t\}$ in order to match as close as possible some moments of the data. Table 3 lists the long run cyclical moments for Colombia and simulated second moments. All data variables are reported in percent deviations from HP filtered trend, except current account and the trade balance, which are reported as a share of GDP. All model variables are reported in percent deviation from ergodic mean except the current account and the trade balance that are reported, as in the data, as share of GDP. To calculate the model moments we simulate the model for 15000 time periods, and retain the final 10000 simulation periods to calculate moments and identify financial crisis events.

The model describes data reasonably well and its results are in line with the business cycle facts for emerging economies. In particular, the model roughly matches the ranking of data volatilities. Consumption volatility is higher than GDP volatility, although both are slightly higher than the data. The relative volatility of consumption is 1.05, very similar to that of 1.04 of the data. Consumption is procyclical and has a high autocorrelation. The current account is less volatile than GDP, as shown by the relative standard deviation of 0.97. On the other hand, the trade balance and the current account are highly autocorrelated in the model as in the data, but the correlation of each with GDP is positive, contrary to the well-documented fact which is that both are counter-cyclical in emerging economies. This result appears because the borrowing constraint does not alter consumption smoothing enough to generate such a negative correlation. Indeed, as shown by Backus et al. (1993), a model with investment would generate a negative correlation. The real exchange rate has a standard deviation lower than that observed in the data, and its correlation with GDP is positive.

As in Mendoza (2010), the simulations are also used to evaluate the model's ability to

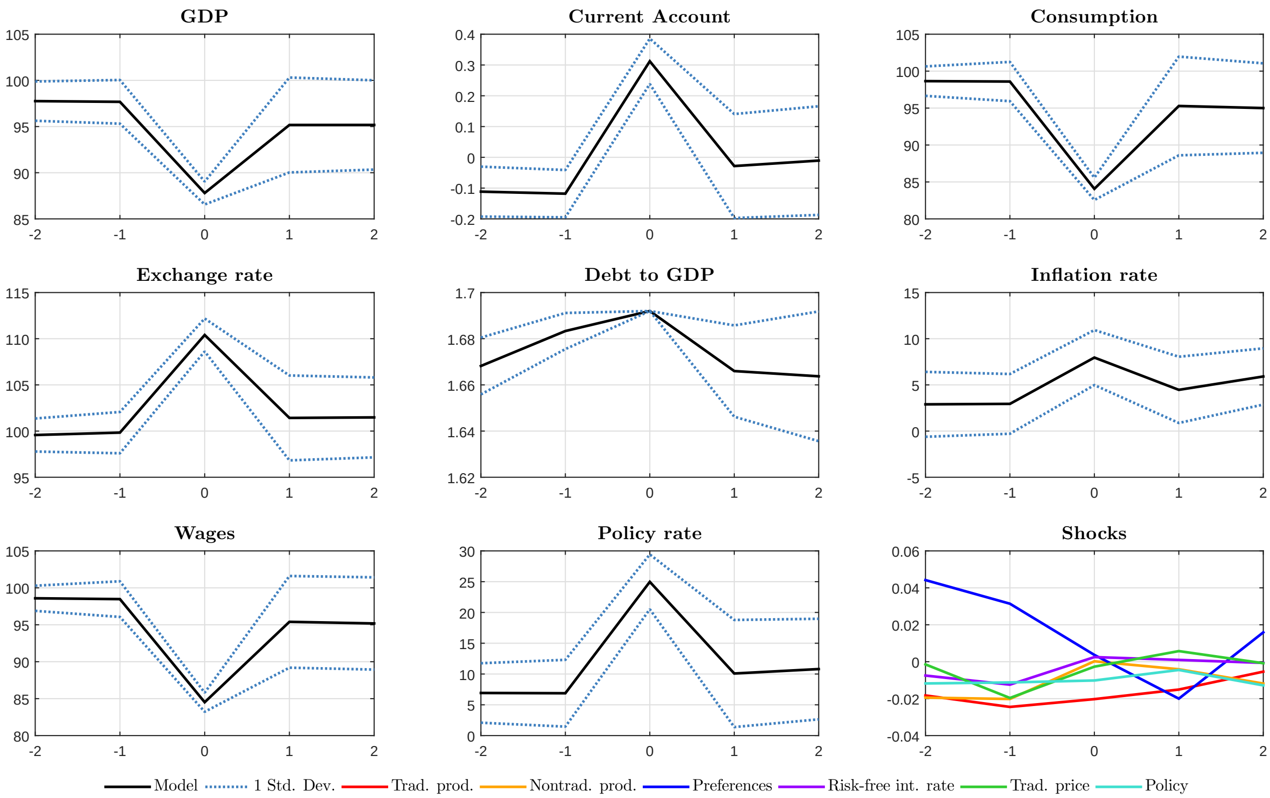
Table 3: Statistical Moments: Model vs the Data

Variable	**Standard dev. in percent**	**Standard. dev. relative to GDP**	**Correlation with GDP**	**First order autocorr.**
Panel A: Colombian data				
GDP	2.60	1.00	1.00	0.76
Consumption	2.70	1.04	0.89	0.75
Current Account	2.20	0.85	-0.34	0.70
Trade Balance	4.60	1.77	-0.39	0.92
Real Exchange Rate	4.40	1.69	-0.71	0.84
Panel B: Model outcome				
GDP	2.84	1.00	1.00	0.71
Consumption	2.98	1.05	0.81	0.67
Current Account	2.75	0.97	0.32	0.57
Trade Balance	1.82	0.64	0.22	0.82
Real Exchange Rate	2.34	0.82	0.27	0.58

Notes: Statistical moments for Colombian data was taken from Hamann et. al. (2015). In this article, data was logged and detrended with the Hodrick-Prescott filter. The data are for the period 1994Q1:2016Q1

account for the dynamics of financial crises. To this end, we use the 10000-period stochastic time series simulation and use the simulated data to construct five-year event windows centered on financial crisis events, where the event is defined in the following way: borrowing constraint does not bind in the first two periods $t = -1, -2$; the borrowing constraint binds at period $t = 0$, GDP is at least one standard deviation below trend, and current account is at least one standard deviation above trend; and there are no restrictions on the borrowing constraint in the last two periods $t = 1, 2$. Figure 2 shows the windows for GDP, consumption, current account, exchange rate, debt to GDP ratio, inflation rate, policy interest rate, wages and the shocks. Each window includes the median across financial crises events identified in the simulation. We also include for comparison one-standard deviation bands.

Figure 2: Financial crisis event windows in model simulations



Notes: Event analysis in the competitive equilibrium with strict inflation targeting. A typical five-period event window is chosen as: (a) no binding borrowing constraints in the first two periods $t = -2, -1$, (b) the borrowing constraint binds at period $t = 0$, GDP is at least one standard deviation below trend, and current account is at least one standard deviation above trend, and (c) no restrictions in the last two periods $t = 1, 2$. The events in the figure are an average of all event series in a simulation of 10000 periods.

As we can see, the model qualitative reproduces the large declines in GDP and expenditure on consumption, and the reversal in the current account that we observe in crisis periods. At $t = 0$ there is not variance for the debt to GDP, given that by definition when the crisis occurs the external debt reaches its limit. If the borrowing constraint is suddenly binding, a sharp macroeconomic adjustment follows: income reduction driven, at least, by permanent declines in the productivity of the sectors, cannot be financed by additional borrowing. Therefore real wages and aggregate demand collapse. The economy depletes external debt, which translates into a exchange rate depreciation. In the balance of payments the trade balance improves and the current account deficit turns positive. Output decreases more than absortion and as a result inflation rate rises above the central bank target. In the event nominal interest rate increases not only because inflation rate is high, but also because the long run interest rate jumps.

The model predicts a recovery at dates $t+1$ and $t+2$, and low interest rate and current account running deficits at $t-1$ and $t-2$. However, we observe some lack of persistence in all model variables that generally recover much faster than in the data. In addition to that, the model predicts an increase in total employment at the financial crisis (not reported), driven by a sharp increase in labor supply, which is at odds with the data. This happens because when the access to foreign debt is curtalied, agents seek to increase their hours worked in order to alleviate the borrowing constraint and to smooth consumption.

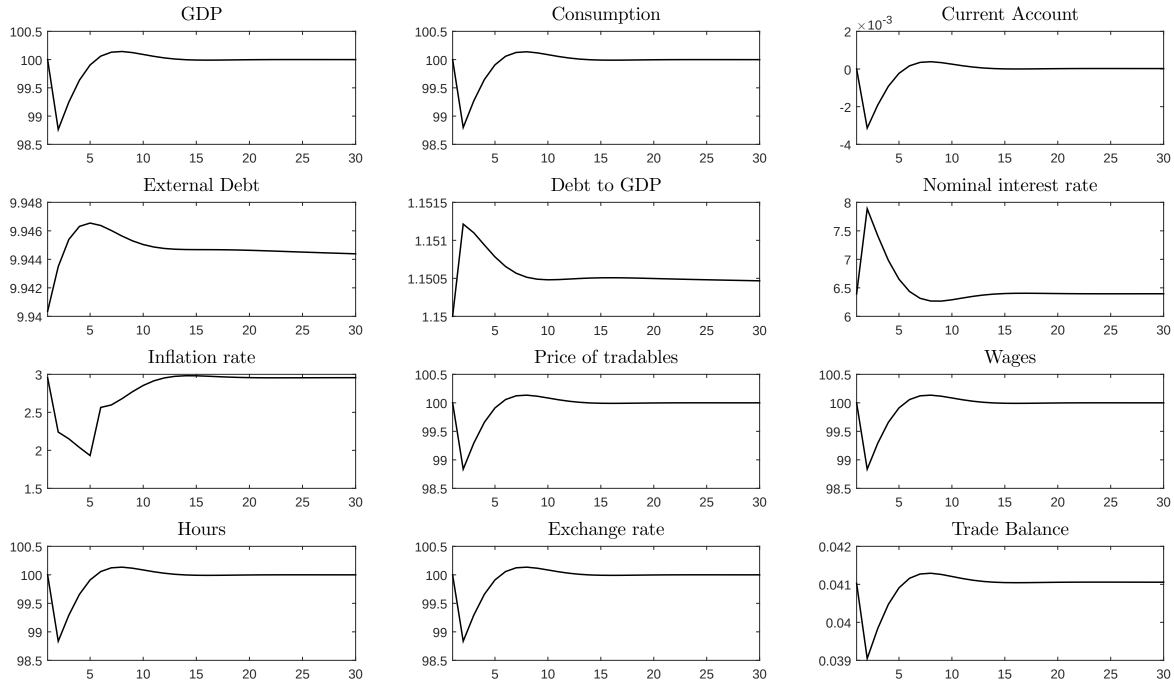
6 Policy Evaluation

In order to complement the quantitative analysis, this section assesses the macroeconomic consequences of adopting alternative Taylor rules. In particular, a nonconventional monetary policy component is considered by including the external debt growth rate as a factor influencing the decision on the interest rate. Then, using the simulation of the previous section, we apply the same draws of the exogenous shocks to each solution of the model that results from modifying the parameters of the Taylor rule. This allows for a fair comparison, since differences in results are almost exclusively attributable to the change in the way the central bank makes its policy decisions.

We assume that the response coefficient for inflation gap is 2.5, as used in several models in Colombia. Then, we modify the values of the response coefficients for the GDP growth rate and the external debt growth rate, using a 5-point discrete grid for both Ω_y and Ω_b on the interval $[0, 2.5]$ and $[-2.5, 0]$ respectively. The reason why Ω_y is positive is obvious, but the sign of Ω_b deserves an explanation. We must identify the effect of the movements of the policy rate on the level of the external debt, and for this, it is useful the impulse-response function (IRF) of a monetary policy shock. The IRFs for the main macroeconomic variables are depicted in Figure 3.

The shock causes nominal interest rate to rise immediately. This leads, according to the UIP, to an exchange rate appreciation, and by PPP a reduction in the relative price of tradable goods. In the labor market the real wage falls, and for this reason, the labor supply in both sectors is reduced. In the nontradable sector, prices fall not only because real marginal cost falls, but because demand contracts. This happens because the increase in the interest rate leads the agents to consume less today. Since consumption is lower and the exchange

Figure 3: Impulse response to a 10% monetary policy shock

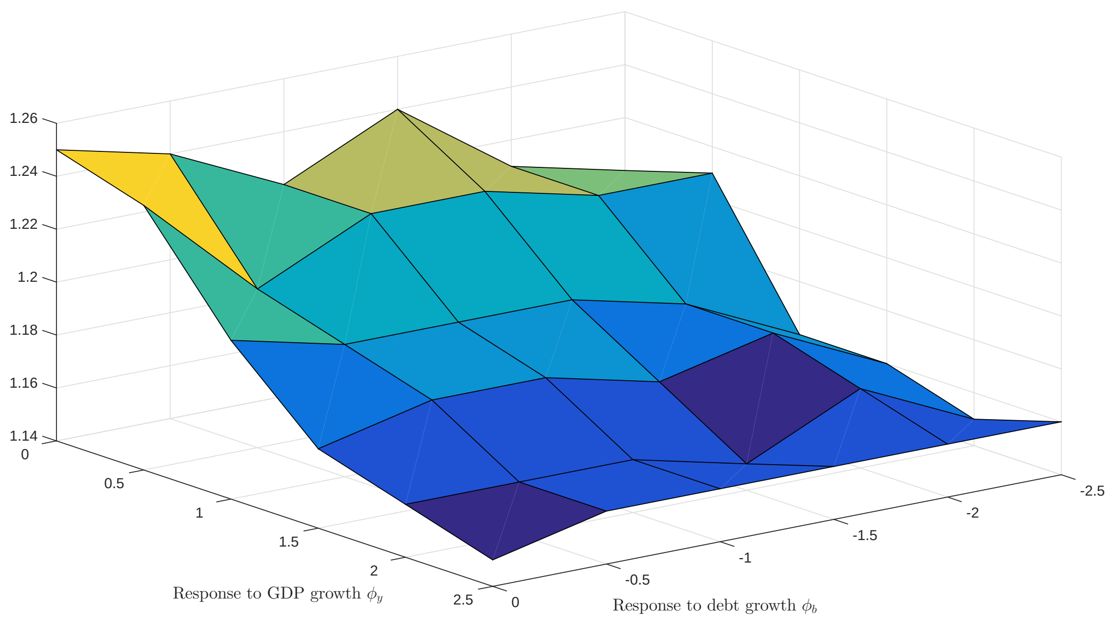


Notes: The figure shows the impulse response functions of GDP, consumption, current account, external debt, debt to GDP, nominal interest rate, inflation rate, price of tradable goods, wages, hours, exchange rate and trade balance to a 10 percent monetary policy shock.

rate appreciation generates a drop in net exports, GDP also falls. The strong response of the GDP relative to the inflation rate is a reflection of the magnitude of nominal rigidities in the model. Added to this, given that domestic financing is more expensive, households increase their relatively low-cost foreign debt stock. Note that the shock is not strong enough to trigger a financial crisis, as the level of indebtedness remains far from the borrowing limit, despite the reduction in GDP.

Then, when the nominal interest rate increases the external debt also does. Therefore, Ω_b must be negative. This means that when debt is rising the nominal interest rate must fall to ensure that the level of indebtedness falls, and possibly, the risk of a financial crisis. However, these reductions in the interest rate also lead to an increase in the inflation rate (as shown in Figure 3), and as a consequence, this nonconventional component in the Taylor rule may threaten price stability. The first question that the model can help to answer is that if the external debt is included in the policy rule, the frequency with which financial crises occur decreases. For this, Figure 4 shows the percentage of time that the economy faces financial crises for each one of the Taylor rules.

Figure 4: Percentage of time in crisis

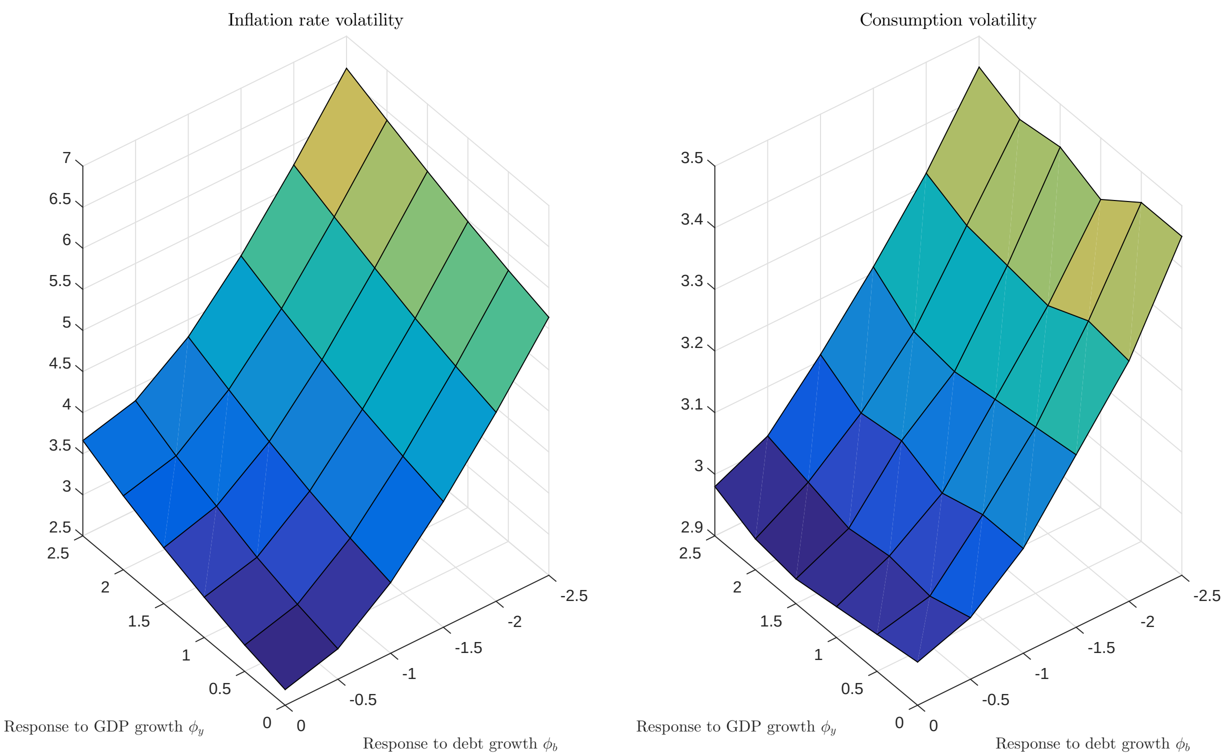


Looking at the figure, it follows that monetary policy decisions have a quantitatively small effect on the fraction of time the economy goes through in crisis. The results show that as the central bank responds more aggressively to increases in debt we do not observe significant reductions in the occurrence of financial crises. For each of the possible coefficients of response to the GDP growth gap, increases in the degree of debt response do not show a decreasing trend, but neither does an increasing one. The model suggests that the unconventional component in the policy rule does not help to prevent financial crises. Also, as the central bank gives more importance to the GDP growth gap when deciding the movements in the interest rate, the percentage of time in crisis falls monotonically. Although in the latter case there is clearly a downward trend, the reduction is also not of a significant magnitude.

If we consider the case in which the external debt is excluded from the Taylor rule, it is observed that when the monetary authority also does not respond to movements in the GDP growth rate, the economy lasts 1.25% of the time in financial crises, while when it responds with a coefficient of 2.5, this percentage decreases to 1.15%. This tends to favor the idea that, although movements in the interest rate can contribute to preventing financial crises, the change in monetary policy required to make a significant difference would be very severe.

Now we go through the next questions: given that agents seek to maintain smooth paths of consumption, what are the consequences of adopting each Taylor rule on consumption volatility? When we incorporate the GDP growth gap or the external debt growth gap into the factors that affect the behavior of the central bank, is the price stability achieved with the IT scheme threatened? To answer this, Figure 5 shows the consumption volatility and the inflation rate volatility that results from the simulation of the model for each Taylor rule.

Figure 5: Inflation and consumption volatilities



One feature stands out from Figure 5: there is a strong and positive relationship between the inflation rate and consumption volatilities, and the degree of responsiveness of the policy interest rate to the external debt growth. As we can see, the volatility of inflation is lower under the current IT scheme (with $\Omega_y = 0$ and $\Omega_b = 0$). The model shows a volatility of 2.7 for this case, and it grows as the coefficient of response to the GDP growth gap increases and the foreign debt growth gap increases. This suggests that any deviation from such a scheme puts at risk the fundamental purpose for which it was created: It was expected, above all, to serve to stabilize medium-term inflation expectations.

Note that the inflation rate variance is higher as the central bank is more concerned about the debt, compared to the case in which it gives more importance to GDP. Then, for the monetary authority, it is more complicated to maintain price stability when, with the same policy instrument, it must contribute to close the current account deficit. And although the same conclusion applies when it is concerned about the GDP growth gap, the trade-off is more evident if one takes into account the result of Figure 4: adopting this policy tends to reduce the frequency of financial crises.

Now, consumption volatility does not show any trend as Ω_y is larger: for the case in which the central bank does not respond to external debt, the volatility is 2.97 when Ω_y is zero, and 2.98 when Ω_y is 2.5 ; That is, there is no great difference as the monetary authority is more concerned about the GDP growth gap. The same is not true for the external debt. Decreases in the debt response coefficient Ω_b translate into greater volatility of consumption. In a scenario with $\Omega_b = 0$, the consumption volatility that results from the model is 3.45 when it responds to the debt with a coefficient of -2.5, which is clearly higher when comparing with the case when the central bank does not take into account the debt on its policy decision.

7 Concluding Remarks

In the last decades several emerging countries have faced financial crises. These events are characterized by significant declines in income, consumption, exchange rate depreciation and current account reversals, etc. These consequences on the business cycle have attracted the attention of academics and policymakers. In particular, central banks, that currently guide their monetary policy decisions based on the IT scheme, have shown their willingness to avoid macroeconomic imbalances, unsustainable external debt levels and the occurrence of financial crises. But actually, justifying a different monetary policy can be difficult since we cannot observe the possible gains in terms of a lower frequency and severity of crises. If interest rates are systematically kept lower than those implied by price stability, inflation expectations can rise and the IT regime could lose credibility. Motivated by this dilemma, in this paper we analyze the macroeconomic implications of adapting the current framework used to structure monetary policy deliberations in a way that takes account of the possibility to prevent the occurrence of financial crises.

In order to address the concern rised above, we examine the macroeconomic performance of different Taylor rules in a regime-switching dynamic stochastic general equilibrium model that describes a small open economy. Specifically, we construct a Newkeynesian model with tradable and nontradable goods, where the nontradable sector faces nominal rigidities. It allows the possibility of occurrence of financial crises through an international borrowing constraint states that households can not borrow over a fraction of their current total income. On the other hand, the central bank decides the movements in the policy rate according to a Taylor rule that responds to the usual factors (inflation gap and product gap), and also adds an unconventional component, by including the gap of external debt growth rate.

The model is calibrated for Colombian data. Quantitative analysis shows that the model describes the data reasonably well and its results are in line with the business cycle facts for emerging economies. Consumption volatility is slighly higher than GDP volatility and current account is less volatile than GDP. Consumption is procyclical and highly autocorrelated and trade balance is also highly autocorrelated. However, it produces lower trade balance and exchange rate volatilities relative to the data. Trade balance and current account are procyclical, contrary to the well-documented fact that both are counter-cyclical in emerging economies. The model also does well at matching the dynamics of financial crises events. Financial crises are preceded by periods of relatively high levels of GDP and absortion, and external deficits, followed by large recessions and reversals in the external accounts when financial crises happen, and then followed by a recovery.

The findings of this paper have important policy implications. First, the nonconventional component of the Taylor rule does not contribute to preventing financial crises. Specifically, as the central bank is more concerned about the external debt growth rate, the occurrence of financial crises does not show a decreasing trend. This does not happen when the central bank gives more importance to the GDP growth gap, since as this happenss the frequency with which crises occur decreases. However, this reduction is quantitatively small. Second, there is a strong and positive relationship between the inflation rate and consumption volatilities, and the degree of responsiveness of the policy interest rate to the external debt growth rate. Assigning an additional objective to the same policy instrument leads to a loss of effectiveness. This means putting at risk at the fundamental purpose of IT: keeping medium-term inflation expectations at anchor to ensure price stability. By introducing new difficulties to achieve the inflation target, the central bank may lose credibility, without significant gains in terms of lower frequency of financial crises.

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A Optimal price setting in the Calvo model

$$\max_{p_t^N(j)} \Pi_t = \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \Delta_{t,t+i} \left[\frac{P_{t+i}^N(j)}{P_{t+i}^N} Y_{t+i}^N(j) - \varphi_{t+i} Y_{t+i}^N(j) \right]$$

subject to:

$$Y_t(j) = \left(\frac{P_t^N(j)}{P_t^N} \right)^{-\theta} Y_t^N$$
$$P_{t+i}^N(j) = P_t^N(j) \left(\prod_{s=1}^{s=i} \left(1 + \pi_{t+s-1}^N \right) \right)$$

Including the constraints into the objective function we obtain:

$$\begin{aligned} \max_{p_t^N(j)} \Pi_t = \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left(\frac{P_t^N(j) \left(\prod_{s=1}^{i} (1 + \pi_{t+s-1}^N) \right)}{P_{t+i}^N} \right)^{1-\theta} Y_{t+i}^N \\ - \epsilon^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} \varphi_{t+i} \left(\frac{P_t^N(j) \left(\prod_{s=1}^{i} (1 + \pi_{t+s-1}^N) \right)}{P_{t+i}^N} \right)^{-\theta} Y_{t+i}^N \end{aligned}$$

Taking the first order condition:

$$\begin{aligned} (\theta - 1) \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \left(P_t^N(j) \right)^{-\theta} \left(\frac{\left(\prod_{s=1}^{i} (1 + \pi_{t+s-1}^N) \right)}{P_{t+i}^N} \right)^{1-\theta} Y_{t+i}^N \\ = \theta \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} \left(P_t^N(j) \right)^{-(1+\theta)} \left(\frac{\left(\prod_{s=1}^{i} (1 + \pi_{t+s-1}^N) \right)}{P_{t+i}^N} \right)^{-\theta} Y_{t+i}^N \end{aligned}$$

By definition:

$$P_{t+i}^N = P_t^N \prod_{s=1}^{i} (1 + \pi_{t+s}^N) \tag{36}$$

Then, using (36) we have that:

$$P_t^N(j) = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} (P_t^N)^{\theta} \left(\prod_{s=1}^{i} \frac{(1+\pi_{t+s-1}^N)}{(1+\pi_{t+s}^N)} \right)^{-\theta} Y_{t+i}^N}{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} (P_t^N)^{\theta-1} \left(\prod_{s=1}^{i} \frac{(1+\pi_{t+s-1}^N)}{(1+\pi_{t+s}^N)} \right)^{1-\theta} Y_{t+i}^N}$$

$$P_t^N(j) = \left(\frac{\theta}{\theta - 1}\right) \frac{P_t^N \mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} \left(\prod_{s=1}^{i} \frac{(1+\pi_{t+s-1}^N)}{(1+\pi_{t+s}^N)}\right)^{-\theta} Y_{t+i}^N}{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} \left(\prod_{s=1}^{i} \frac{(1+\pi_{t+s-1}^N)}{(1+\pi_{t+s}^N)}\right)^{1-\theta} Y_{t+i}^N}$$

Finally:

$$\hat{p_t^N}(j) = \left(\frac{\theta}{\theta - 1}\right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \varphi_{t+i} \left(\frac{1+\pi_{t+i}^N}{1+\pi_t^N}\right)^{\theta} Y_{t+i}^N}{\mathbb{E}_t \sum_{i=0}^{\infty} \epsilon^i \beta^i \lambda_{t+i} \left(\frac{1+\pi_{t+i}^N}{1+\pi_t^N}\right)^{\theta-1} Y_{t+i}^N}$$

B The complete model

$$C_t = \left(C_t^T\right)^{\omega} \left(C_t^N\right)^{1-\omega}$$

$$C_t^T = \frac{\omega C_t}{p_t^T}$$

$$C_t^N = \frac{(1-\omega)C_t}{p_t^N}$$

$$\left(z_t^u C_t - \frac{H_t^{\delta}}{\delta}\right)^{-\rho} = \lambda_t$$

$$H_t^{\delta-1} = \left(1 + \kappa \frac{\mu_t}{\lambda_t}\right) w_t$$

$$e_t \lambda_t = \beta \mathbb{E}_t (1 + r_t^{\star}) e_{t+1} \lambda_{t+1} + e_t \mu_t$$

$$\lambda_t = \beta \mathbb{E}_t (1 + r_t) \lambda_{t+1}$$

$$1 + r_t = \left(\frac{1 + i_t}{1 + \pi_{t+1}}\right)$$

$$Y_t^T = z_t^T H_t^T$$

$$Y_t^N = z_t^N H_t^N$$

$$w_t = p_t^T z_t^T$$

$$w_t = \varphi_t p_t^N z_t^N$$

$$H_t = H_t^T + H_t^N$$

$$p_t^T = e_t p_t^{T\star}$$

$$\hat{p_t^N}(j) = \left(\frac{\theta}{\theta - 1}\right) \mathbb{E}_t \left(\frac{\Theta_t}{\Psi_t}\right)$$

$$\Theta_t = \lambda_t \varphi_t Y_t^N + \epsilon \beta \left(\frac{1 + \pi_{t+1}^N}{1 + \pi_t^N}\right)^{\theta} \Theta_{t+1}$$

$$\Psi_t = \lambda_t Y_t^N + \epsilon\beta \left(\frac{1+\pi_{t+1}^N}{1+\pi_t^N}\right)^{\theta-1} \Psi_{t+1}$$

$$1 = \epsilon \left(\frac{1+\pi_{t-1}^N}{1+\pi_t^N}\right)^{1-\theta} + (1-\epsilon)\hat{p_t^N}(j)^{1-\theta}$$

$$\frac{p_t^N}{p_{t-1}^N} = \frac{1+\pi_t^N}{1+\pi_t}$$

$$C_t^N = Y_t^N$$

$$Y_t = C_t + (1+r_{t-1}^\star)e_t b_{t-1}^\star - e_t b_t^\star$$

$$r_t^\star = r_t^f + \Phi\left[\exp\left(\frac{e_t b_t^\star}{Y_t} - \Upsilon\right) - 1\right]$$

$$i_t = \rho^i i_{t-1} + (1-\rho^i)\left[\bar{r} + \bar{\pi} + \Omega_\pi(\pi_t - \bar{\pi}) + \Omega_y(y_t - \bar{y}) + \Omega_b(\hat{b_t^\star} - \bar{\hat{b^\star}})\right] + \epsilon_t^i$$

$$\xi(s_t)\tilde{b_t^\star} = (1-\xi(s_t))\,\mu_t$$

$$z_t^u = \rho^{z^u} z_{t-1}^u + \left(1-\rho^{z^u}\right)\log\left(\bar{z^u}\right) + \sigma^{z^u}\epsilon_t^{z^u}$$

$$z_t^T = \rho^{z^T} z_{t-1}^T + \left(1-\rho^{z^T}\right)\log\left(\bar{z^T}(s_t)\right) + \sigma^{z^T}\epsilon_t^{z^T}$$

$$z_t^N = \rho^{z^N} z_{t-1}^N + \left(1-\rho^{z^N}\right)\log\left(\bar{z^N}(s_t)\right) + \sigma^{z^N}\epsilon_t^{z^N}$$

$$r_t^f = \rho^{r^f} r_{t-1}^f + \left(1-\rho^{r^f}\right)\log\left(\bar{r^f}\right) + \sigma^{r^f}\epsilon_t^{r^f}$$

$$p_t^{T\star} = \rho^{p^{T\star}} p_{t-1}^{T\star} + \left(1-\rho^{p^{T\star}}\right)\log\left(\bar{p^{T\star}}\right) + \sigma^{p^{T\star}}\epsilon^{p^{T\star}}$$